MATH 151 Help Session Problems

1. A box has a rectangular bottom, with one side three times as long as the other, and an open top. The total area of the material used to make the box is 144cm². What is the largest possible volume the box can have? What are the dimensions of the box with the largest volume?

2. A ladder 15 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 3 m/s, how fast is the angle between the top of the ladder and the wall changing when the angle is π/6 radians?

3. Prove \( \frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}} \)

4. Evaluate \( \lim_{x \to 0} \left( \frac{1}{x^2} - \frac{1}{x \sin x} \right) \).

5. Differentiate \( f(x) = \log_3 \sin \left( e^{-3x} \right) \).

6. Find the tangent line for
   (a) \( f(x) = (x^2 + 1) \sin(2x) \) at \( x = \frac{\pi}{2} \)
   (b) \( x = t^2, y = t^3 + 9t \) at \( t = 1 \).

7. If \( \overrightarrow{r}(t) = (\arctan(2t), \ln(t + 1)) \)
   (a) Find a tangent vector at \( t = \frac{1}{2} \).
   (b) Find a vector equation of the tangent line at \( t = \frac{1}{2} \).
   (c) Find the angle between the position and tangent vector at \( t = \frac{1}{2} \).

8. If \( f(x) = e^{-x^2} \)
   (a) find where \( f(x) \) is increasing and decreasing and identify local extrema.
   (b) find concavity and inflection points.

9. \( \int \cos^4 \theta \sin \theta d\theta \)

10. A ball is thrown upward with an initial velocity of 64 feet per second from an initial height of 80 feet. (Assume the gravitational force on the ball is 32 ft/s².)
    (a) Find the position function that gives the height \( s \) as a function of time \( t \).
    (b) When does the ball hit the ground?