2021 Power Team
Texas A& M High School Mathematics Contest
November 13, 2021

In the following problems you will be asked to find a strategy of finding an object using balance scale. It is assumed that one can put sets of objects (coins, weights, etc.) on both pans of the scale and see whether the sets have equal weight or one set is heavier than the other, and see which one is heavier. You will be asked to describe a strategy to find something and the minimal number of steps in the strategy. Here "minimal number of steps" means the smallest number of steps you need in the *worst case*. You are supposed to justify that it is the minimal number of steps (i.e., that one can not find the object using smaller number of steps in all cases).

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Problem 1. Alice chose an integer from 1 to 100. Bob wants to find this number by asking Alice "yes or no" questions. Find the smallest number of questions Bob can ask Alice so that he will for sure guess the number chosen by Alice.

Problem 2. Suppose that we have 9 coins: 8 of the same weight, and one fake coin visually indistinguishable for the other 8 but lighter. What is the smallest number of weighings on a balance scale needed to find the fake coin?

Problem 3. Find a formula for the number of weighings needed to find the fake coin if there are $N-1$ coins of one weight and one lighter fake coin.

Problem 4. There are 8 coins. We know that either all of them have the same weight, or one is fake (lighter). What is the smallest number of weighings you need to find the fake coin or to prove that it does not exist?

Problem 5. We have one golden coin, 3 silver coins, and 5 brass coins. Coins made of different metals are visually different and have different weight. One of the coins is fake (lighter than its legal weight). What is the smallest number of weighings needed to find the fake coin?

Problem 6. We have 12 golden coins and 12 silver coins. One of the coins is fake. A fake silver coin is lighter than a genuine silver coin. A fake golden coin is heavier than a genuine golden coin. You are not allowed to put more than 4 coins of the same metal on one pan of the scale. What is the smallest number of weighings needed to find the fake coin?

Problem 7. We have 9 indistinguishable coins, one of which is fake (lighter than the other coins). We have three balance scales, one of which is broken (it produces random unpredictable results not related to the actual weights on the pans). We don't know which coin is fake and which scale is broken. Find the fake coin by 4 weighings.

Problem 8. We have 20 visually indistinguishable coins of two kinds: x coins of weight a each and $20 - x$ coins of weight b each, where $b > a$. Find x using 11 weighings.

Problem 9. We have 13 masses of 1, 2, 3, …, 13 oz (labeled by their weights). But exactly one of the masses is defective (its weight is different from the label, but it is not known if it is heavier or lighter). What is the smallest number of weighings needed to find the wrong mass?

Problem 10. We have 10 masses of 1, 2, 3, …, 10 oz, labeled by stickers with their weights written on them. However, two stickers for masses different by 1 oz (i.e., n and $n + 1$ for some n) were switched. What is the smallest number of weighings needed to find switched stickers?