Modeling Phytoplankton with Switching and in Riverine Systems

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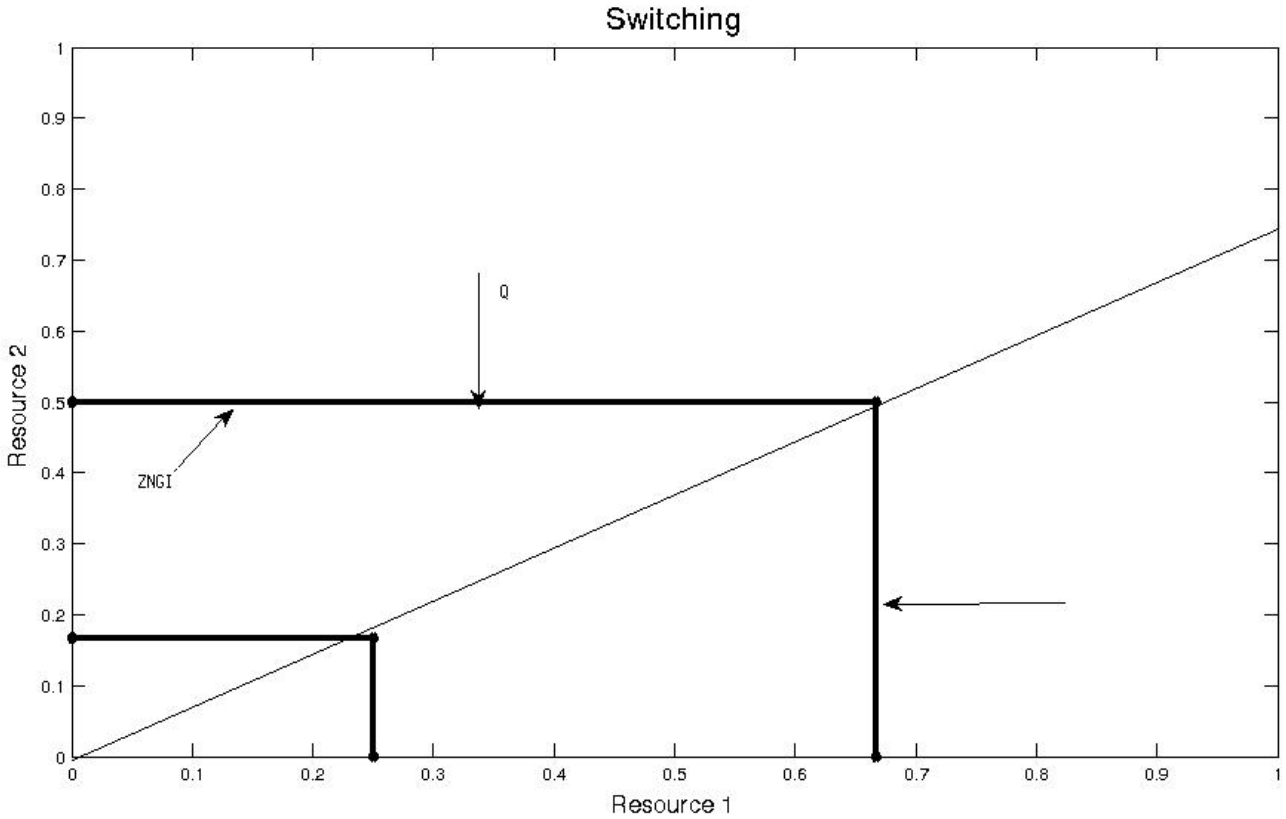
Outline

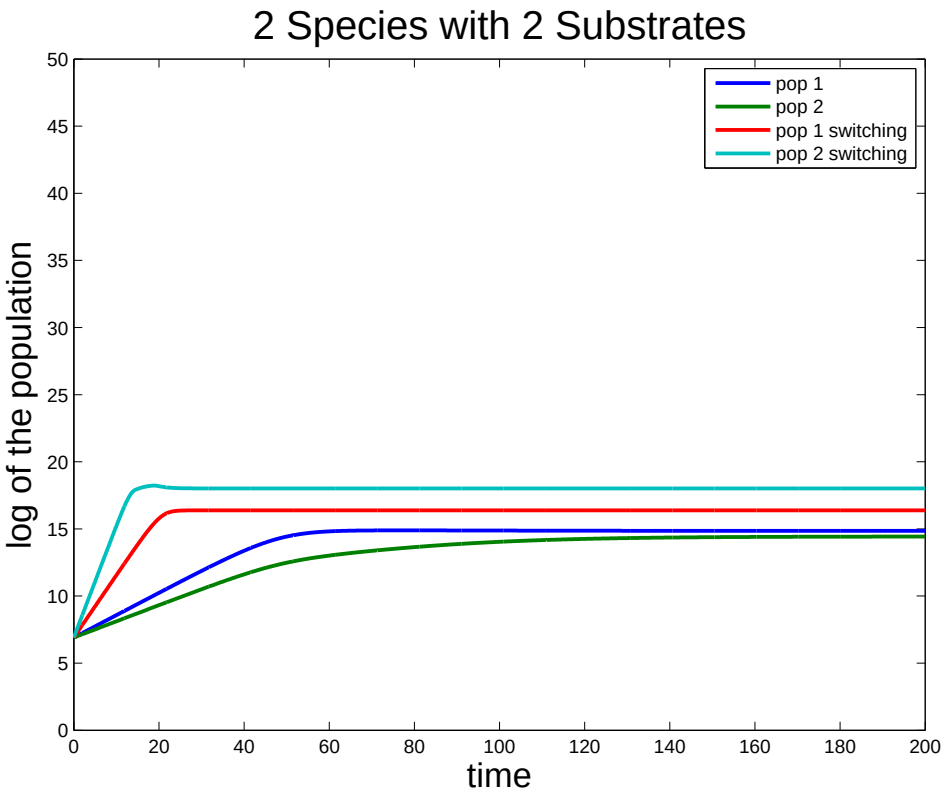
Recap

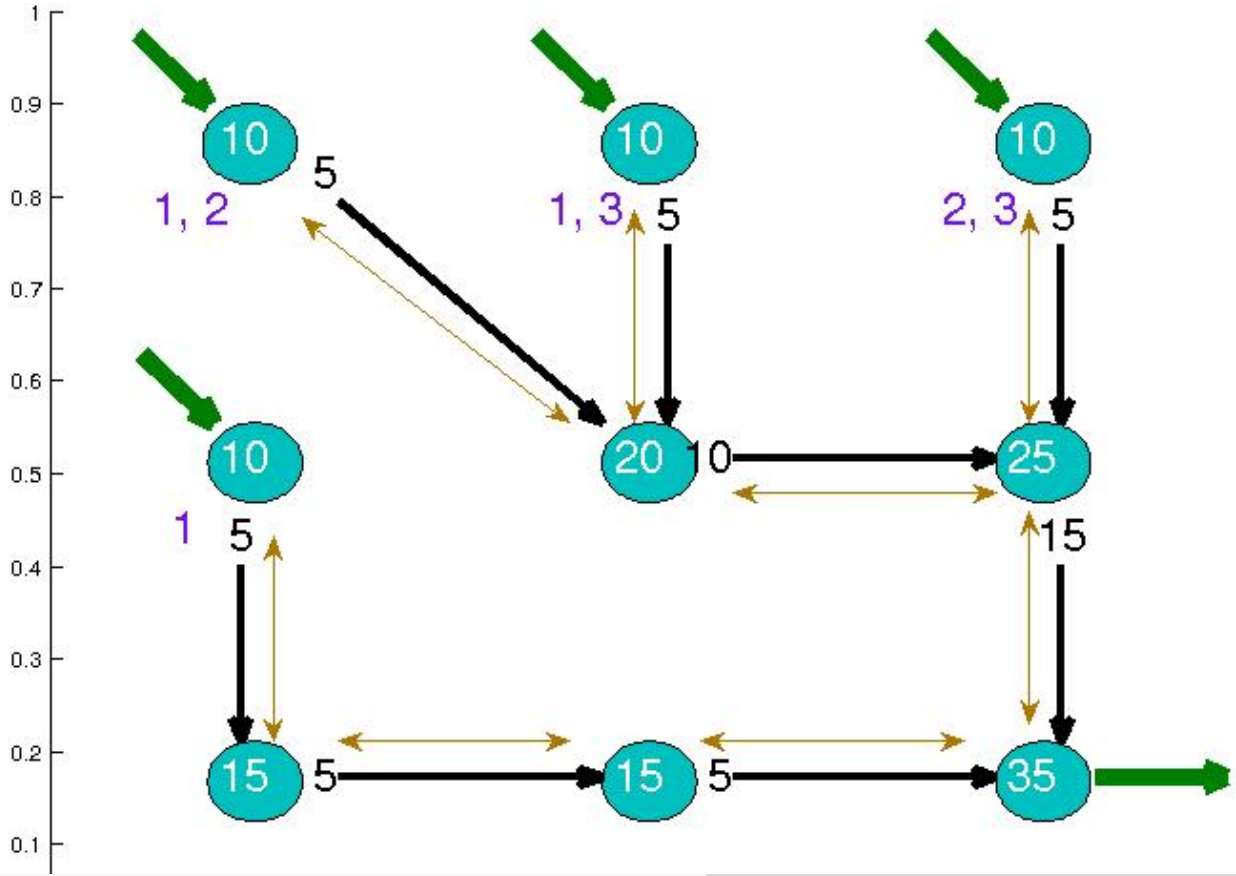
- What are phytoplankton?
- Different types of nutrient intake (essential, switching, substitutable, etc.)
- Modeling phytoplankton population growth when limited by essential nutrients


Switching
Q
ZNGI
Resource 2
Resource 1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Switching Model

- Population (no change)
  $\frac{dN_i}{dt} = \mu_i N_i - \nu N_i$
- Substrate (Switching) $\frac{dS_j}{dt} = \nu(S_{in} - S_j) - \sum_{i=1}^{n} h(S_j)\mu_i N_i$
- Resource Dependent Growth Equation (Switching)
  $\mu_i = \tilde{\mu}_i \max_j \left(\frac{S_j}{S_j + \kappa_{ij}} \right)$
- $h(S_j) = \begin{cases} Q_{ij} & \text{if } \frac{S_j}{S_j - k_{ij}} \text{ is the max} \\ 0 & \text{otherwise} \end{cases}$


2 Species with 2 Substrates
log of the population
time
pop 1
pop 2
pop 1 switching
pop 2 switching
0
5
10
15
20
25
30
35
40
45
50
0
20
40
60
80
100
120
140
160
180
200


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
10
1, 2
5
10
1, 3
5
10
2, 3
5
10
1
5
20
10
25
15
15
5
15
5
35

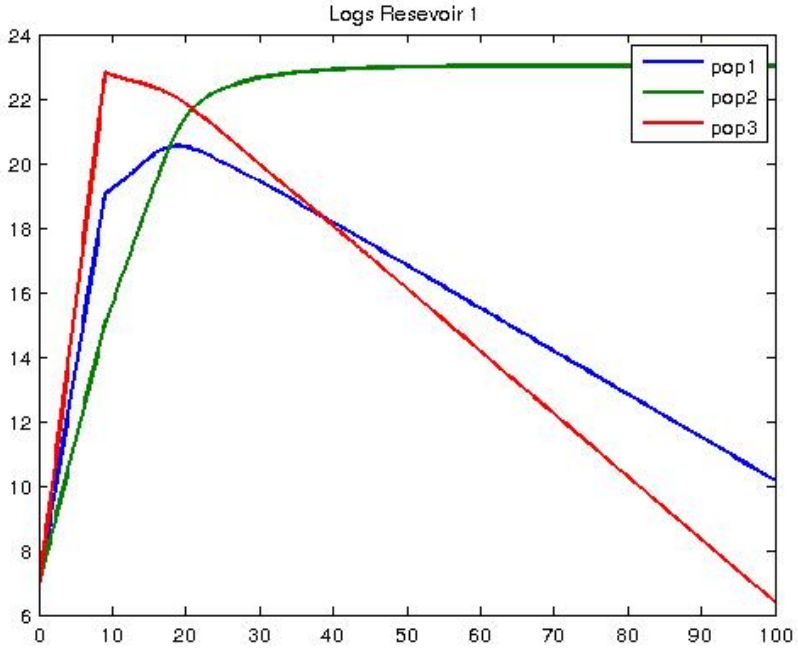
- **For each species in each resevoir:** Population change = population growth − flow out of system − population deaths − flow out within system + flow in within system
- $$\frac{dN_{ik}}{dt} = \left(\mu_{ik} - \frac{f_k}{V_k} - m_i - \sum_{n=1}^{z} \frac{D_{kn}}{V_k}\right) N_{ik} + \sum_{n=1}^{z} \left(\frac{D_{nk}}{V_n} N_{in}\right)$$
-

$$i := \text{species}$$

$$k := \text{resevoir}$$

$$z := \text{total number of resevoirs}$$

- **For each resource in each resevoir:** Nutrient change = flow in from outside system − flow out of system − flow out within system + flow in within system − nutrient consumed
- $$\frac{dS_{jk}}{dt} = \left(F_k - \frac{f_k}{V_k} - \sum_{n=1}^{z} \frac{D_{kn}}{V_k}\right) S_{jk} + \sum_{n=1}^{z} \left(\frac{D_{nk}}{V_n} S_{jn}\right) - \sum_{n=1}^{r} Q_{nj} \mu_{nk} N_{nk}$$
-
$$j := \text{nutrient}$$
$$k := \text{resevoir}$$
$$z := \text{total number of resevoirs}$$
$$r := \text{total number of species}$$



$$\tilde{\mu} = [1.5 \quad 1 \quad 2]$$

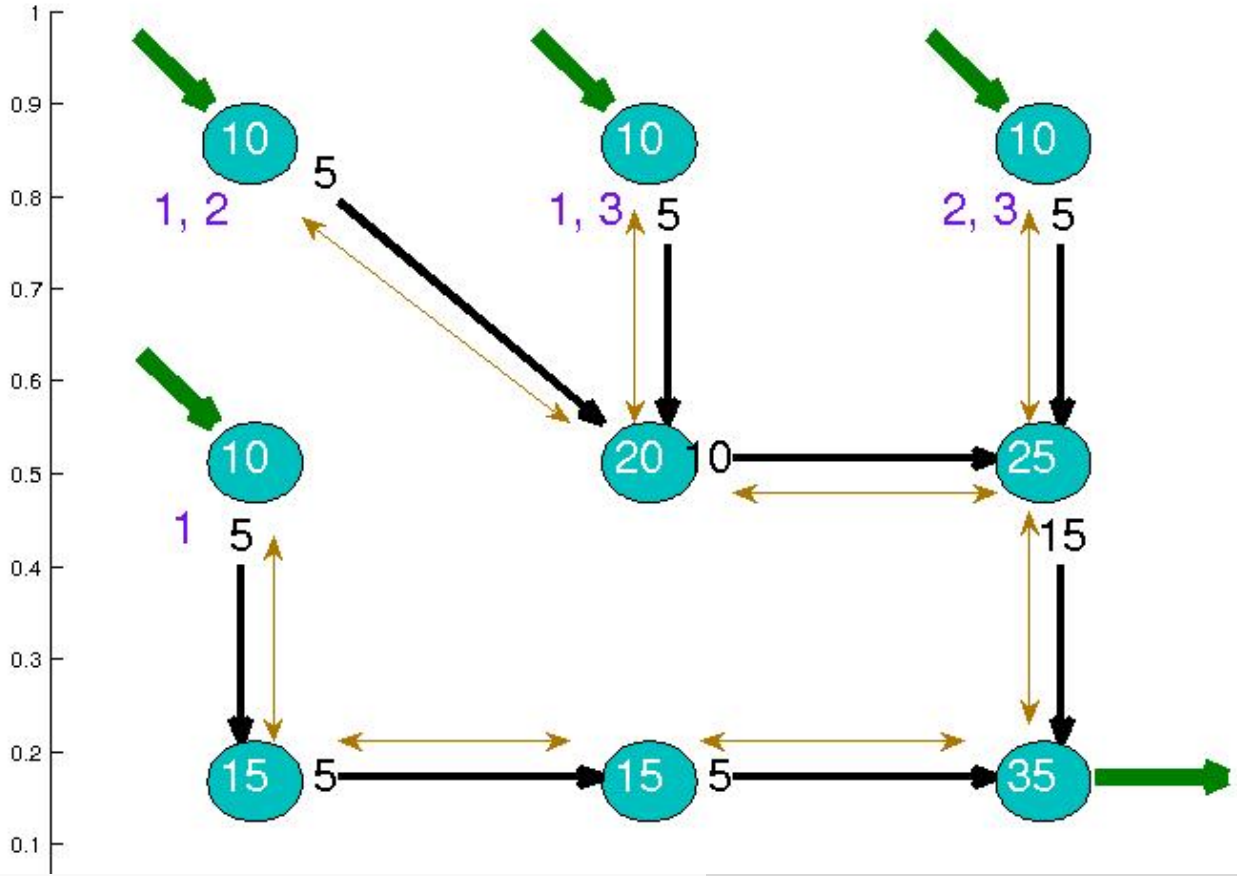
$$m = [.15 \quad .1 \quad .2]$$

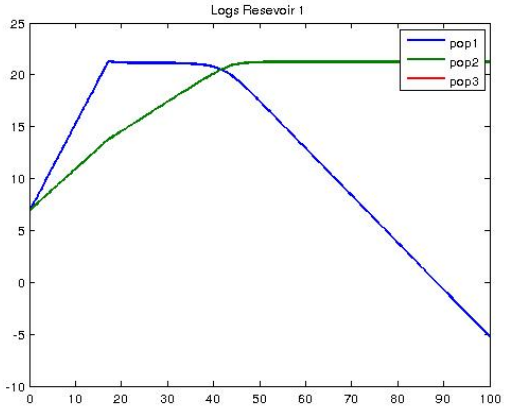
$$\kappa = \begin{bmatrix} .5 & .05 & 2 \\ .25 & .005 & .1 \end{bmatrix}$$

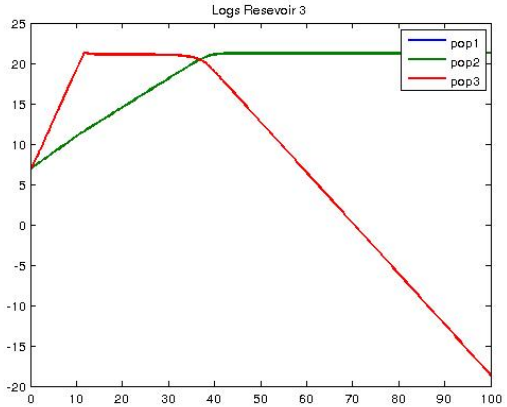
$$\tilde{S} = \begin{bmatrix} 10 \\ 7.5 \end{bmatrix}$$

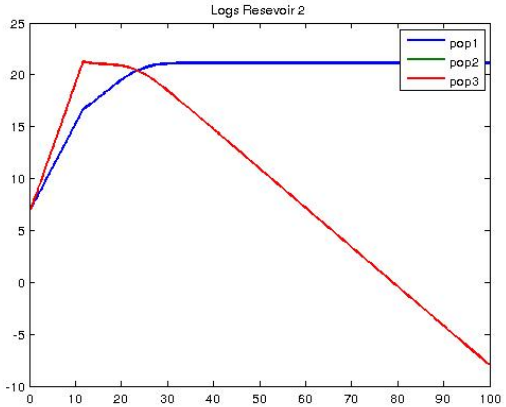
$$\tilde{N} = [1000 \quad 1000 \quad 1000]$$

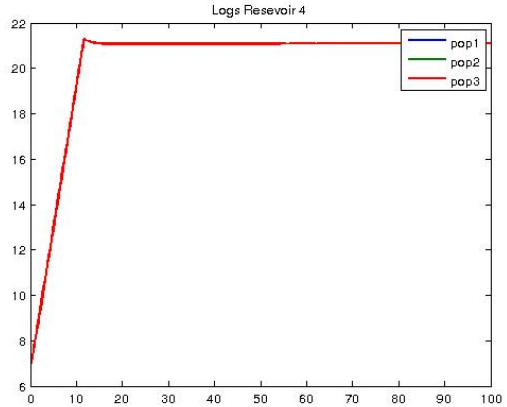
$$Q = \begin{bmatrix} 50 & 50 & 50 \\ 25 & 5 & 2.5 \end{bmatrix} * 10^{-9}$$

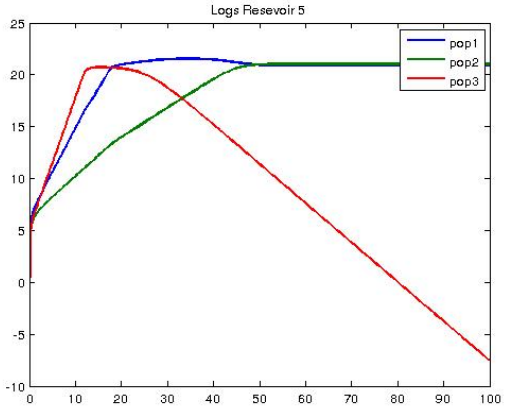

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
10
1, 2
5
10
1, 3
5
10
2, 3
5
10
1
5
20
10
25
15
15
5
15
5
35

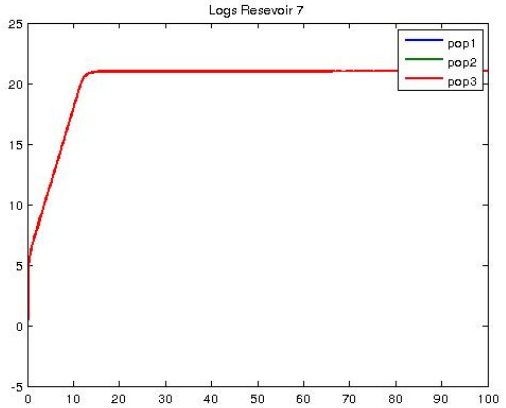

Logs Resevoir 1
pop1
pop2
pop3

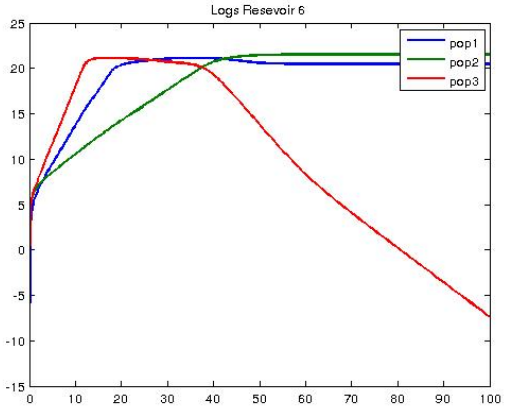

Logs Resevoir 3
pop1
pop2
pop3

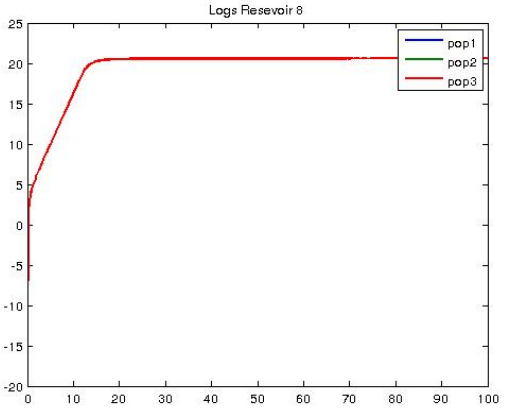

Logs Resevoir 2
pop1
pop2
pop3

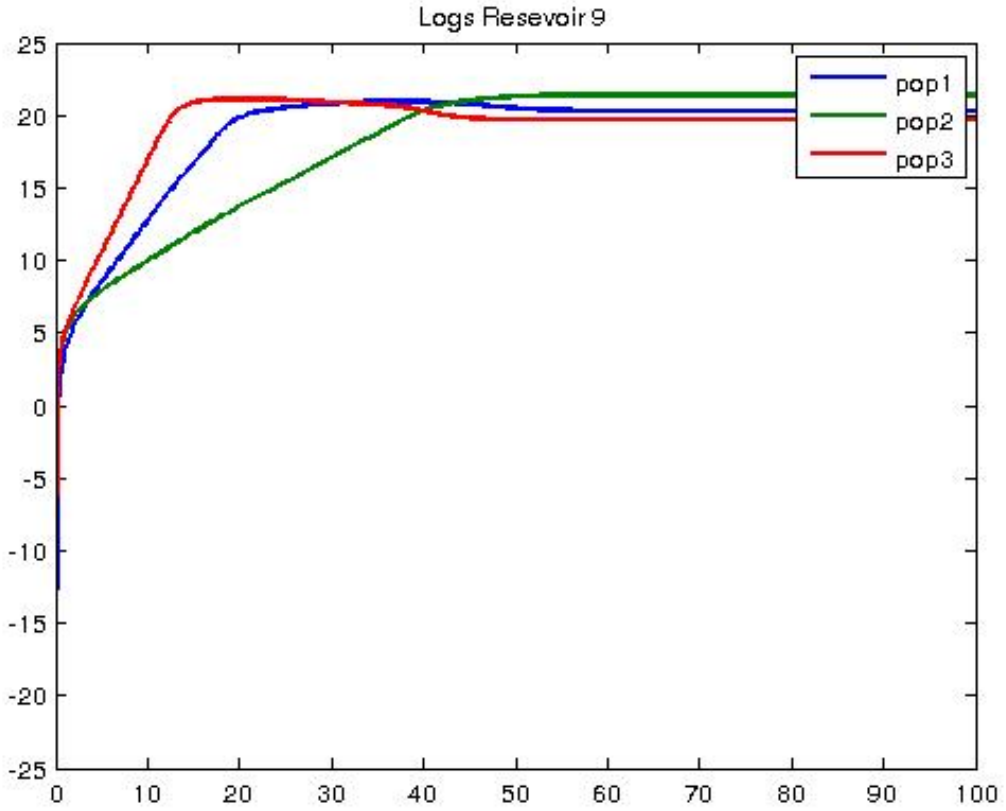

Logs Resevoir 4
pop1
pop2
pop3

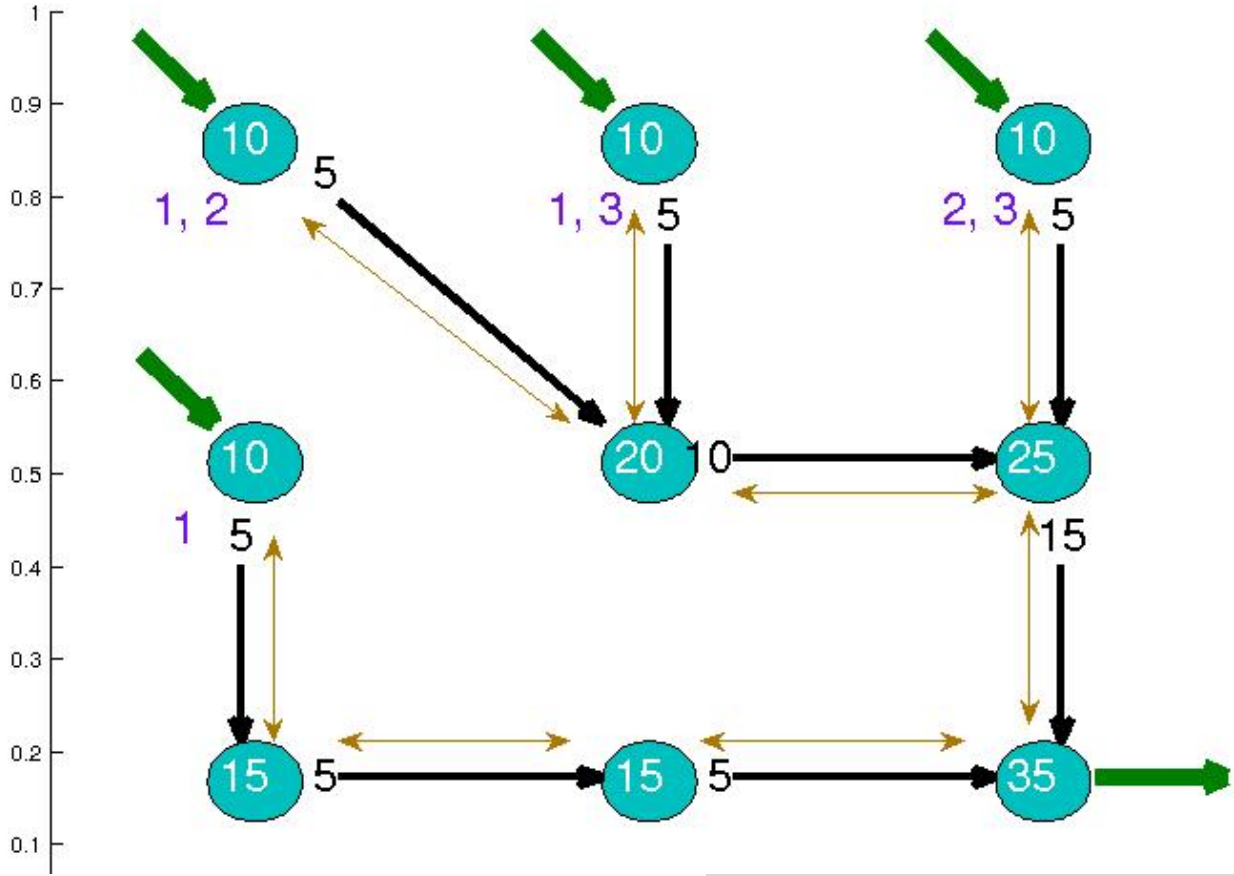

Logs Resevoir 5
pop1
pop2
pop3


Logs Resevoir 7
pop1
pop2
pop3


Logs Resevoir 6
pop1
pop2
pop3


Logs Resevoir 8
pop1
pop2
pop3


Logs Resevoir 9
pop1
pop2
pop3
25
20
15
10
5
0
-5
-10
-15
-20
-25
0
10
20
30
40
50
60
70
80
90
100


1
0.9
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0.4
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0.1
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1, 2
5
10
1, 3
5
10
2, 3
5
10
1
5
20
10
25
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15
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15
5
35

- **For each species in each resevoir:**
  Population change = population growth − flow out of system − population deaths − flow out within system + flow in within system + diffusion in − diffusion out
- $$\frac{dN_{ik}}{dt} = \left(\mu_{ik} - \frac{f_k}{V_k} - m_i - \sum_{n=1}^{z} \frac{D_{kn}}{V_k}\right) N_{ik} + \sum_{n=1}^{z} \left(\frac{D_{nk}}{V_n} N_{in}\right) + \sum_{n=1}^{z} \frac{E_{kn}}{V_n} N_{ik} - \sum_{n=1}^{z} \frac{E_{nk}}{V_k} N_{ik}$$
-

$$i := \text{species}$$

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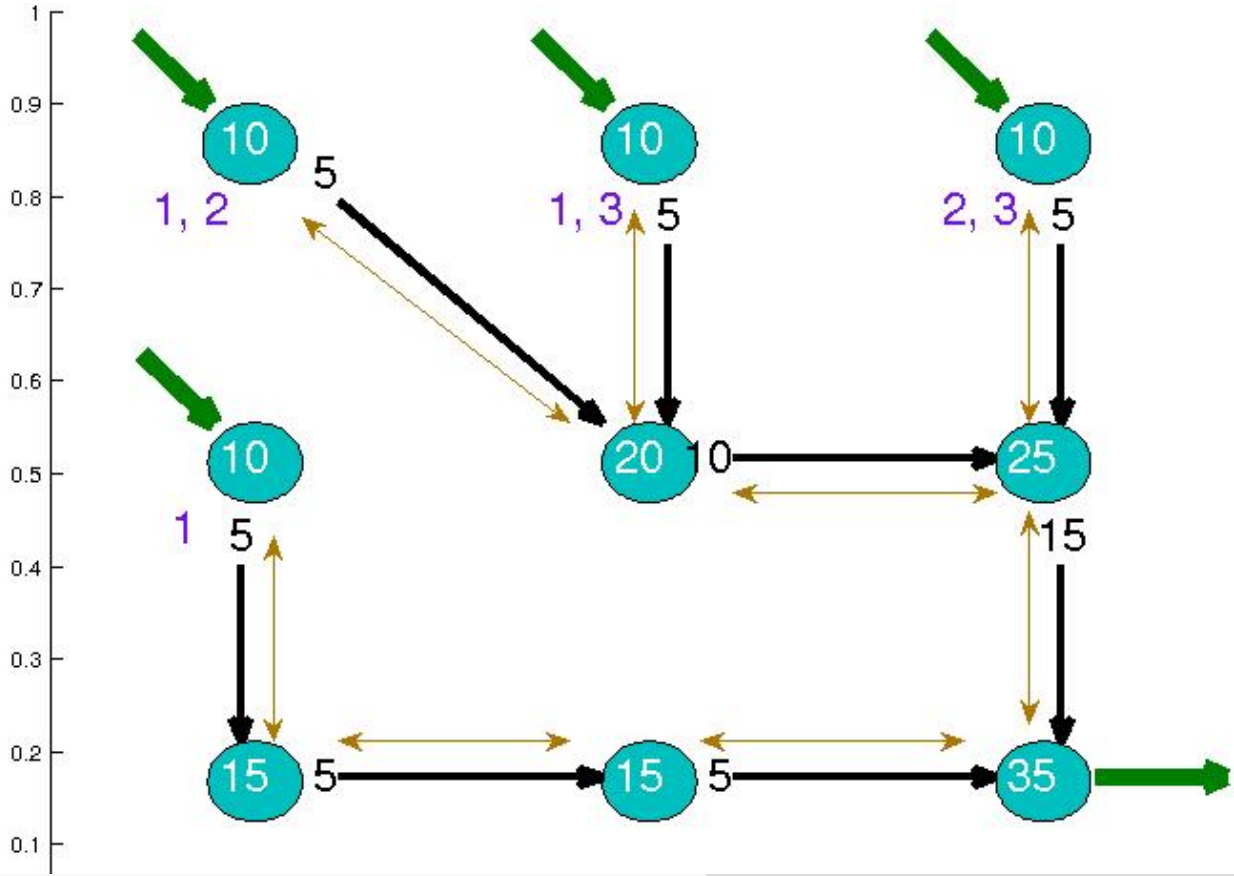
- **For each resource in each resevoir:** Nutrient change = flow in from outside system − flow out of system − flow out within system + flow in within system − nutrient consumed + diffusion in − diffusion out
- $$\frac{dS_{jk}}{dt} = \left(F_k - \frac{f_k}{V_k} - \sum_{n=1}^{z} \frac{D_{kn}}{V_k}\right) S_{jk} + \sum_{n=1}^{z} \left(\frac{D_{nk}}{V_n} S_{jn}\right) - \sum_{n=1}^{r} Q_{nj} \mu_{nk} N_{nk} + \sum_{n=1}^{z} \frac{E_{kn}}{V_n} S_{jk} - \sum_{n=1}^{z} \frac{E_{nk}}{V_k} S_{jk}$$
-

$j :=$ nutrient

$k :=$ resevoir

$z :=$ total number of resevoirs

$r :=$ total number of species


1
0.9
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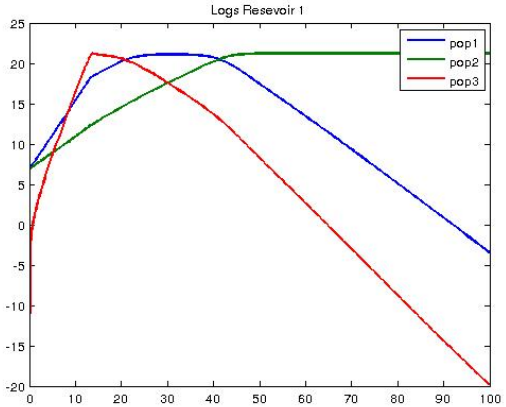
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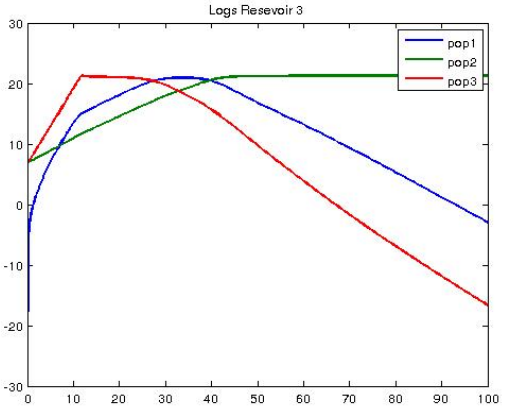
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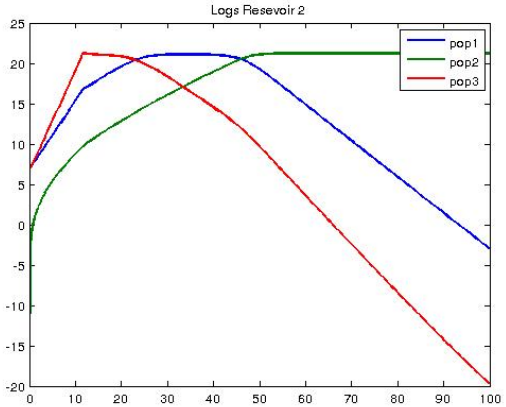
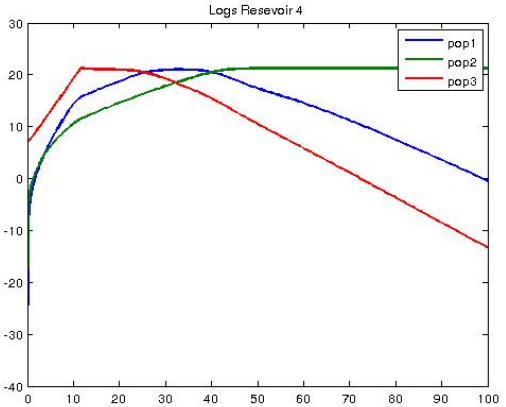
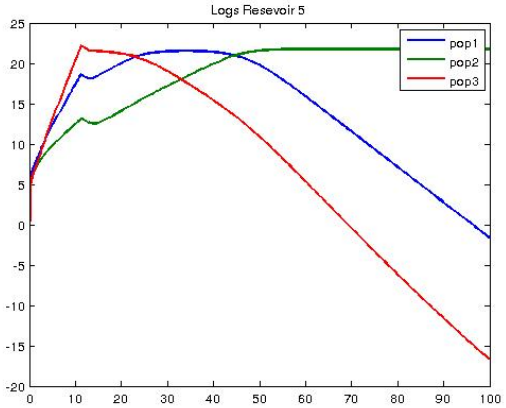

Logs Resevoir 2
pop1
pop2
pop3


Logs Resevoir 4
pop1
pop2
pop3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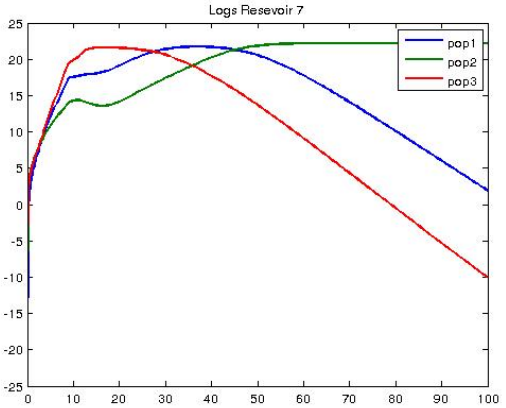
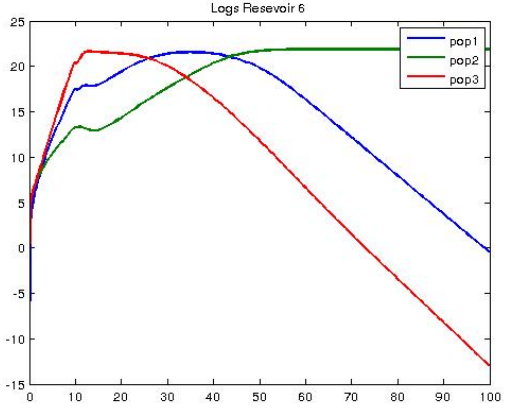
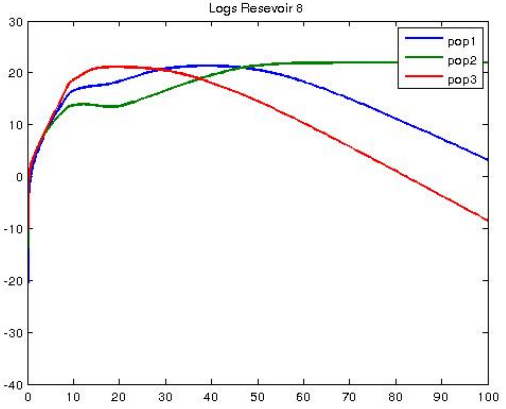
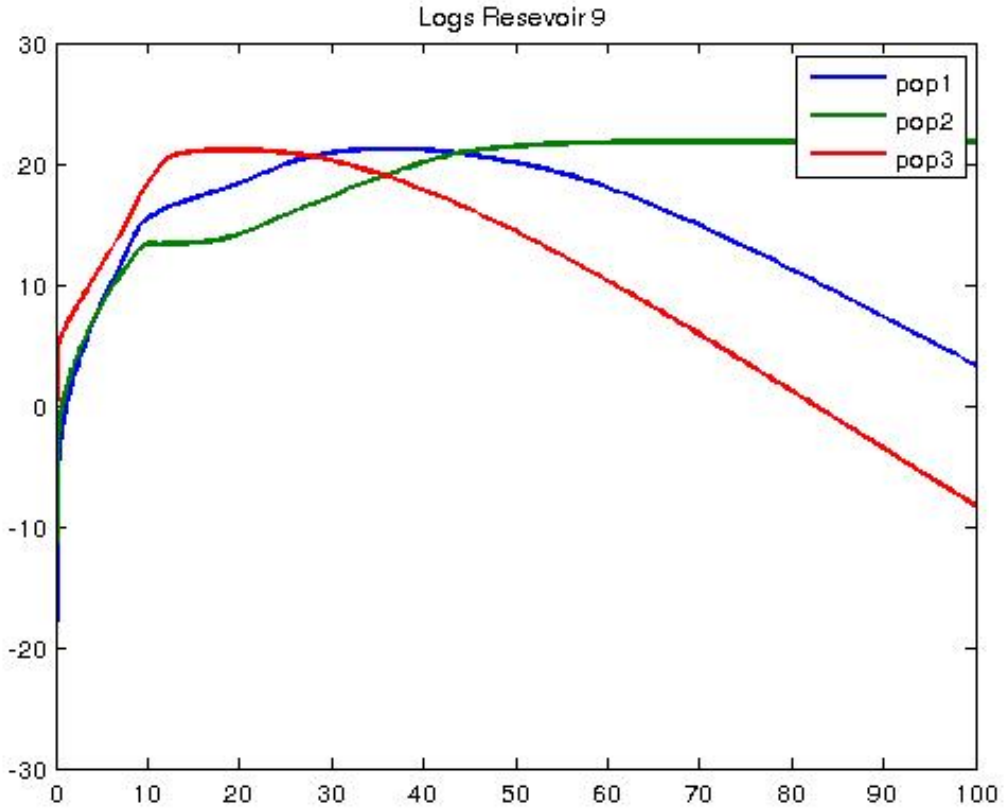
pop2
pop3


Logs Resevoir 7
pop1
pop2
pop3


Logs Resevoir 6
pop1
pop2
pop3


Logs Resevoir 8
pop1
pop2
pop3


Logs Resevoir 9
pop1
pop2
pop3
30
20
10
0
-10
-20
-30
0
10
20
30
40
50
60
70
80
90
100