Math142 Lecture Notes
2.6 - Continuity and Nondifferentiability

**Definition of Continuity**

A function, \( f \), is said to be continuous at the point \( x = a \) if all the following are true.

1. \( f(a) \) is defined.
2. \( \lim_{{x \to a}} f(x) \) exists.
3. \( \lim_{{x \to a}} f(x) = f(a) \).

(A continuous function has no holes, gaps, or breaks in its graph.)

**Example 1:** Find all points of discontinuity in the graphs below. If \( f \) is not continuous at a point, state which of the three conditions given in the definition of continuity is violated.

Points of Discontinuity:  
Points of Discontinuity:  
Points of Discontinuity:

**Example 2:** Determine the continuity of \( f(x) = \frac{x - 4}{x^2 - 16} \) at the indicated points.

(a) at \( x = 4 \)  
(b) at \( x = -4 \)  
(c) at \( x = 0 \)
Example 3: Determine the continuity of \( g(x) = \begin{cases} 
  x - 3 & x \leq 0 \\
  x^2 - 3 & 0 < x \leq 1 \\
  x & x > 1 
\end{cases} \) at (a) \( x = 0 \) (b) \( x = 1 \)

Example 4: Find the value of \( k \) that will make \( g(x) = \begin{cases} 
  -x + 2k & x \geq 4 \\
  2x & x < 4 
\end{cases} \) continuous on \((-\infty, \infty)\).

<table>
<thead>
<tr>
<th>Functions that are Continuous on ((-\infty, \infty))</th>
<th>Functions that are Continuous on their domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Polynomial</td>
<td>• Natural Logarithm</td>
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<tr>
<td>• Exponential</td>
<td>• Radical (Rational Exponent)</td>
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<tr>
<td></td>
<td>• Rational (discontinuous at values of ( x ) that make the denominator 0).</td>
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</table>
Example 5: Determine the intervals where the following functions are continuous.

(a) \( f(x) = e^{0.5x} \)

(b) \( f(x) = 4 - \ln(2 - 8x) \)

(c) \( f(x) = \sqrt{5x - 15} \)

(d) \( f(x) = \frac{4x - 5}{(4x - 5)(x + 2)} \)

(e) \( g(x) = 4x^2 - 4x + 1 \)

A function is not differentiable at \( x = c \) if the graph of
the function

- has a sharp turn or corner (cusp) at \( x = c \).
- has a vertical tangent at \( x = c \) \([f'(c) \text{ is undefined}].\)
- is not continuous at \( x = c \).

Example 4: For what values of \( x \) is the function graphed below not differentiable?

Example 5: Determine where \( f(x) = |x + 2| - 3 \) is not differentiable.