Math142 Lecture Notes
4.1 - The Chain Rule

**Chain Rule**
If $y = f(u)$ and $u = g(x)$ are used to define $h(x)$, where $h(x) = f(g(x))$, then

$$h'(x) = f'(g(x)) \cdot g'(x)$$

**Generalized Power Rule**
If $u$ is a differentiable function of $x$ and $n$ is any real number with $f(x) = [u(x)]^n$ then

$$f'(x) = n[u(x)]^{n-1} \cdot u'(x)$$

**The Derivative of a Mess**
If you have an expression of the form $y = (\text{mess})^n$, where $n$ is any real number, the derivative is

$$y' = n(\text{mess})^{n-1}(\text{mess}')$$

Example 1: Differentiate the following functions.

(a) $f(x) = (8 - 3x)^5$

(b) $g(x) = 4(7x^2 - 5)^3$

(c) $f(x) = \frac{6}{x^3 - 4x + 8}$

(d) $f(x) = \frac{9}{\sqrt{2x^3 + 4x}}$
Example 2: Find the following. You may need to use more than one derivative rule.

(a) Find $f'(x)$ if $f(x) = x(5 - 2x)^3$

(b) \[ \frac{d}{dx} \left( \frac{x^3}{(2x^3 - x + 4)^3} \right) \]

(c) Find the equation of the line tangent to $g(x) = x^2 \sqrt{3x^2 - 11}$ at $x=2$. 

Example 3: During its first season, the number of viewers for *Traveling with Teachers* can be modeled by

\[ v(w) = (12 + 8w)^{\frac{2}{3}} \]

where \( w \) is the number of weeks since the show began, and \( v \) is the number of viewers in hundred thousands. Evaluate and interpret \( v'(10) \)

Example 4: The Catchy Logo Company which does advertising for newly released films reports the revenue for a new film can be modeled by

\[ R(x) = 3(x^2 + 24)^{\frac{1}{3}} \]

where \( x \) is the number of thousands of dollars spent on advertising, and \( R(x) \) is the revenue in millions of dollars.

(a) Find \( MR(4) \) and interpret.

(b) Find \( AR(4) \) and interpret.

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