Math142 Lecture Notes

4.2 - Derivatives of Logarithmic Functions

<table>
<thead>
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<th>Derivative Rules for Logarithmic Functions</th>
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<tr>
<td>• For $f(x) = \ln x$, with $x &gt; 0$, $f'(x) = \frac{d}{dx}(\ln x) = \frac{1}{x}$.</td>
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<td>• If $g$ is a differentiable function of $x$ and the range of $g$ is $(0, \infty)$, the derivative of $h(x) = \ln[g(x)]$ is $h'(x) = \frac{g'(x)}{g(x)}$.</td>
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<tr>
<td>• If $g$ is a differentiable function, where the range of $g$ is $(0, \infty)$, then the derivative of $f(x) = \log_b[g(x)]$ is $f(x) = \frac{g'(x)}{g(x) \cdot \ln b}$.</td>
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Example 1: Differentiate and simplify (unless otherwise stated) the following.

(a) $f(x) = 5 - 2\ln x$

(b) $f(x) = \left(\frac{1}{\sqrt{x^2}}\right)(\ln x)$

(c) $f(x) = \ln (2x^3 - 7)$
(d) \( f(x) = (\ln x)^3 \)

(e) \( f(x) = [\ln(x^4 + 5x)]^2 \)

(f) \( f(x) = \frac{3x^2 - 5x + 6}{\ln(4x^2 + 1)} \)  \( \text{ (do not simplify) } \)

(g) \( f(x) = \log_8 x \)

(h) \( f(x) = \left[ \log_2(x^5) \right] \left( \log_3 x \right) \)
Example 2: Given \( f(x) = (8x^2)(\ln x) \):

(a) Find the derivative, \( f'(x) \).

(b) Find the equation of the line tangent to the graph of \( f(x) \) at \( x = 2 \).

Example 3: The average expenditure for a new domestic car in the United States can be modeled by

\[
 f(x) = 15.302 + 1.685 \ln(x) \quad 1 \leq x \leq 7
\]

where \( x \) represents the number of years since 1980 and \( f(x) \) represents the average expenditure (in thousands of dollars) for a new domestic car.

(a) Find the derivative, \( f'(x) \).

(b) Find the equation of the line tangent to the graph of \( f(x) \) when \( x = e \).
Example 4: Note the difference in finding the derivative of each of the following functions:

(a) \( f(x) = \ln x^3 \)

(b) \( g(x) = (\ln x)^4 \)

Summary: Derivative Rules for Logarithmic Functions

- \( y = \ln x, \) with \( x > 0, \) \( y' = \frac{1}{x}. \)
- \( y = \ln(\text{mess}), \) with \( \text{mess} > 0, \) \( y' = \frac{\text{mess}'}{\text{mess}}. \)
- \( y = \log_b x, \) and \( x > 0, \) \( y' = \frac{1}{x \cdot \ln b}. \)
- \( y = \log_b(\text{mess}), \) and \( \text{mess} > 0, \) \( y' = \frac{\text{mess}'}{\text{mess} \cdot \ln b}. \)

Note: \( \text{mess} \) is an expression containing \( x \)'s