Multiple Choice

1. 5 different cities are to be connected by straight roads. How many different possible roads are there which connect the 5 cities?

A) 5! B) $5^5$ C) 10 D) P(5,5) E) none of these

Solution: Let the 5 cities be denoted by $C_1$, $C_2$, $C_3$, $C_4$, and $C_5$. Let the road connecting $C_1$ and $C_2$ be denoted by $R_{12}$. Then the possible combinations are: $R_{1,2}$, $R_{1,3}$, $R_{1,4}$, $R_{1,5}$, $R_{2,3}$, $R_{2,4}$, $R_{2,5}$, $R_{3,4}$, $R_{3,5}$, and $R_{4,5}$. There are 10 in all. The correct answer is C.

2. Given the set \{1,2,3,4,5,6,7,8,9\}, how many two-digit numbers are there whose digits increase from left to right (e.g. 12, 56, etc)

A) 50 B) 9 C) 9! D) 36 E) none of these

Solution: The list of two digit increasing numbers constructed from this list is:

\{12,13,14,15,16,17,18,19,  
23,24,25,26,27,28,29,  
34,35,36,37,38,39,  
45,46,47,48,49,  
56,57,58,59,  
67,68,69,  
78,79,  
89\}.

There are 36 combinations in all. There are 36 decreasing, 9 equal (\{11,22,etc.\}). The correct answer is D.
3. The sets $E$ and $E^c$ are always mutually exclusive.

   (A) True    (B) False

   Since $E \cap E^c = \emptyset$, $E$ and $E^c$ are mutually exclusive. The answer is true.

4. The vertices of a feasible set are given by the points $\{(0,5),(1,1),(4,0),(6,6)\}$. The maximum of $P=3x-2y$ is given by

   A) -10    B) 12    C) 6    D) 0    E) none of these

   **Solution:** Evaluating $P$ at each of the vertices, $P(0,5)=-10$, $P(1,1)=1$, $P(4,0)=12$, $P(6,6)=6$. The maximum value is 12, so the correct answer is B.

5. The set $E^c \cup F^c$ is given by

   A) I,III,IV    B) I,II,III    C) I,II    D) II only    E) none of these

   **Solution:** $E^c$ is formed from regions III and IV. $F^c$ is formed from regions I and IV. The union is therefore $\{I, III, IV\}$. The correct answer is A.

**Workout Problems**

6. 4 adults and 2 children (under the age of 12) walk up to a rental car which can hold 6 passengers.

   (a) [10 pts] If all the adults can drive, how many different seating arrangements are there?

   **Solution:** There are $C(4,1)=4$ ways to choose the driver. Of the 5 remaining seats, there are 5 individuals. Since the order of the seating matters (front seat is different than back, etc), there are $P(5,5)=5!$ ways to distribute 5 people into 5 seats. The multiplication principle says that there are therefore $4*5!=4*120=480$ ways to seat the 6 individuals if all 4 adults drive.

   (b) [10 pts] If only 3 adults drive, how many seating arrangements are there?

   **Solution:** This is exactly the same as above, but now there are $C(3,1)P(5,5)=3*120=360$ ways.
7. [10 pts] Given the inequalities:

\[
\begin{align*}
6x + 5y & \leq 60 \\
3x + y & \geq 15 \\
x + y & \geq 8 \\
x & \geq 0 \\
y & \geq 0
\end{align*}
\]

Graph these inequalities, and shade in the feasible set, i.e. the region bounded by the inequalities above. Label the corners by their coordinates.

**Solution:** The feasible set is bounded by the polygon with vertices (corners) at \(\left(\frac{5}{3},10\right)\), \(\left(\frac{7}{2},\frac{9}{2}\right)\), \((8,0)\), \((10,0)\).

8. An investor has a maximum of $100,000.00 to invest in 3 stocks. Stock A has a profit of 20% and a risk of 12%. Stock B has a profit of 15% and a risk of 8%. Stock C has a profit of 10% and a risk of 5%. The average risk is defined by

\[
\frac{r_1s_1 + r_2s_2 + r_3s_3}{s_1 + s_2 + s_3}
\]

where \(r_i\) is the risk, and \(s_i\) is the amount of stock.

The investor wishes to maximize his profit, with a maximum average risk of 10%.

**Solution:** The constraints are \(x + y + z \leq 100,000\) for the total investment. The average risk constraint is given by

\[
\frac{0.12x + 0.08y + 0.05z}{x + y + z} \leq 0.10
\]
(a) [10 pts] Write down the simplex tableau for this problem

Solution: The tableau is given by

\[
\begin{array}{cccccc|c}
 x & y & z & u & v & P & C \\
 1.0 & 1.0 & 1.0 & 1 & 0 & 0 & 100 \\
 0.02 & -0.02 & -0.05 & 0 & 1 & 0 & 0 \\
 -0.2 & -0.15 & -0.10 & 0 & 0 & 1 & 0 \\
\end{array}
\]

(b) [10 pts] Reduce the tableau to standard form

Solution: Apply the Simplex Algorithm to get

\[
\begin{array}{cccccc|c}
 x & y & z & u & v & P & C \\
 0.0 & 1.0 & 1.75 & 0.5 & -25 & 0 & 50 \\
 1.0 & 0.0 & -0.75 & 0.5 & 25 & 0 & 50 \\
 0.0 & 0.0 & 0.0125 & 0.175 & 1.25 & 1 & 17.5 \\
\end{array}
\]

(c) [5 pts] Find the solution to this problem.

Solution: Reading of the solution from part b), we get \( x = 50,000, y = 50,000 \) and \( z = 0 \) with \( P = 17,500 \). The average risk is 0.10.

9. [10 pts] A standard deck of 52 cards is well-shuffled. How many different ways can one have a pair of spades or diamonds?

Solution: Consider the spades: there are 3 cards to choose from 13, and 39 other (non-spade) cards to choose from. This gives \( C(13,3) \times C(39,2) \). A similar argument holds for diamonds. The combinations add together.

Solution: \( 2 \times C(13,3) \times C(39,2) \)

10. [10 points] In the area below, show the following set \((A \cap B) \cup C^c\)

Solution: