1. [10 pts] Section 4.1 # 59
   The cost function for direct materials in a furniture factory was approximated by $y = C(x) = 0.667x - 0.01563x^2 + 0.000151x^3$ for $0 \leq x \leq 50$ where $x$ is output in thousands of dollars. Find the approximate value of $x$ for which marginal cost is at a minimum.
   
   **Solution:** The marginal cost is the derivative of the Cost function, $C(x)$. $C'(x) = 0.667 - 0.03126x + 0.000453x^2$. This is minimized when
   
   $$C''(x) = -0.03126 + 0.000906x = 0$$

   Therefore, $x = 34.503$ Remember, $x$ is in units of 1000’s, so the value is really 34,503 units.

2. [10 pts] Section 5.3 # 9
   If the velocity (as a function of time) is given by $v(t) = \sqrt{t}$ find the total distance travelled on $0 \leq t \leq 5$. Estimate the distance by using the midpoint rule with $N = 5$.
   
   **Solution:** The exact distance is given by the integral
   
   $$\int_0^5 \sqrt{t} \, dt = \frac{2}{3} t^{3/2} \bigg|_{t=0}^{t=5} = 7.45356$$

   The approximate distance (via the midpoint formula) is
   
   $$(\sqrt{0.5} + \sqrt{1.5} + \sqrt{2.5} + \sqrt{3.5} + \sqrt{4.5})(1.0) = 7.50514$$

3. [10 pts] Section 4.1 # 19
   Find all critical values, the largest open intervals on which $f$ is increasing, the largest open intervals on which $f$ is decreasing, and all relative maxima and minima. Sketch a rough graph of $f$.

   $$f = 1 - (1 - x)^{2/3}$$
Solution: The critical points are places where $f' = 0$ or $f'$ is undefined. Since

$$f'(x) = \frac{2}{3}(1 - x)^{-1/2}$$

the only critical point is $x = 1$. $f' > 0$ if $x < 1$ and $f' < 0$ if $x > 1$.

4. [10 pts] Section 4.2 # 59

Find all the critical points and points of inflection for

$$f(x) = x^4 + x^3 + x^2 + x + 1$$

Sketch the curve.

Solution: The critical points and inflection points are found by examining

$$f'(x) = 4x^3 + 3x^2 + 2x + 1 = 0$$

and

$$f''(x) = 12x^2 + 6x + 2 = 0$$

To find the zeros of $f'$ we use the [2nd][calc][zero] command on the TI83. This gives us $x = -0.6058296$ as the critical point. The inflection points are given by the quadratic formula which in this case tells us there are no real roots (square root of a negative number) and therefore no inflection points.
The graph is below

5. [10 pts] Section 5.1 # 47

The height of a ball, initially at \( h_0 \) feet above the ground, thrown with velocity \( v_0 \) feet per second, is given by

\[ h(t) = h_0 + v_0 t - 16t^2 \]

Before it hits the ground, find the maximum velocity, and maximum height, a ball achieves when it is thrown up into the air at a rate of 50 ft/sec from a height of 6 feet.

**Solution:** Substituting the values from the problem

\[ h(t) = 6 + 50t - 16t^2 \]

The maximum height is achieved when \( h'(t) = 50 - 32t = 0 \), or \( t = 50/32 \). At this time, \( h_{max} = h(50/32) = 45.0625 \).

Since the velocity is \( v(t) = h'(t) = 50 - 32t \), we have \( v'(t) = -32 \) which never vanishes. This means the maximum must occur at the end points. When \( t = 0, v = v(0) = 50 \). When it hits the ground, \( h = 0 \) or \( 6 + 50T - 16T^2 = 0 \) or \( T = 3.24 \) seconds. For this time, \( v = v(T) = -53.68 \). The maximum velocity is 53.68 ft/sec.

6. [10 pts] Section 5.2 # 33

Find the indefinite integral

\[ \int \frac{1}{x(\ln x)^2} \, dx \]
Solution: This is solved by a change of variable \( u = \ln x \). In this case, \( du = 1/x \, dx \) so the integral reduces to
\[
\int \frac{du}{u^2} = -\frac{1}{u} + C = -\frac{1}{\ln |x|} + C
\]

7. [10 pts] Section 5.5 # 22
Evaluate the definite integral
\[
\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx
\]
Solution: This is solved by a change of variable \( u = \sqrt{x} \). In this case, \( du = 1/(2\sqrt{x}) \, dx \), and
\[
\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx = \int 2e^u \, du = 2e^u + C = 2e^{\sqrt{x}} + C
\]
so
\[
\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx = 2e^{\sqrt{x}} \bigg|_{x=1}^{x=4} = 2e^2 - 2e^1 = 9.341548541
\]
This can also be done via the TI83 programs prgmRiemann(), and FnInt().

8. [10 pts] Section 4.3 # 18
Locate the value(s) at which each function attains an absolute maximum and an absolute minimum, if they exist, of the given function on the given interval.
\[
f(x) = -8x^3 + 6x - 1 \text{ on } [-1, 1]
\]
Solution: The critical points are found by examining \( f'(x) \). \( f'(x) = -24x^2 + 6 = 0 \) when \( x^2 = 6/24 = 1/4 \) so \( x = \pm 1/2 \). We also look at the endpoints, and conclude \( f(-1) = 1 \), \( f(-1/2) = -3 \), \( f(1/2) = 1 \) and \( f(1) = -3 \). Therefore there are absolute maxima at \( x = -1, 1/2 \) and absolute minima at \( x = -1/2, 1 \).

The graph is below:
9. [10 pts] Section 5.1 # 37

Find the anti-derivative

$$\int \frac{x + 1}{x} \, dx$$

**Solution:** As a general rule, **simplify first** then **change of variables** then integrate. We have

$$\int \frac{x + 1}{x} \, dx = \int (1 + \frac{1}{x}) \, dx = x + \ln x + C$$

10. [10 pts] Section 5.6 # 6

Find the area enclosed by the curves $y = \sqrt{x}$ and $y = x$ on $[0,1]$

**Solution:** This is given by the difference of the two integrals

$$\int_{0}^{1} \sqrt{x} \, dx - \int_{0}^{1} x \, dx = \frac{2}{3}x^{3/2}|_{x=1} - \frac{1}{2}x^{2}|_{x=0} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$