1. [5 pts] Find the rate of change of the function \( f(x) = x^2 + 1 \) at the point \( x = 2 \) by using limits.

**Solution:** By the definition of the derivative, the rate of change of \( f(x) \) at \( x = 2 \) is given by

\[
\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2} \frac{(x^2 + 1) - (5)}{x - 2} = \lim_{x \to 2} \frac{x^2 - 4}{x - 2} = \lim_{x \to 2} (x + 2) = 4
\]

You can also use

\[
\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
\]

2. [10 pts] Find the derivative of the function \( x + \sqrt{x} \) at \( x = 1 \) by using limits.

**Solution:** By the definition of the derivative, the rate of change of \( f(x) \) at \( x = 1 \) is given by

\[
\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{x + \sqrt{x} - 2}{x - 1} = \lim_{x \to 1} \frac{(x - 1) + (\sqrt{x} - 1)}{x - 1}
\]

\[
= \lim_{x \to 1} 1 + \frac{\sqrt{x} - 1}{x - 1} = 1 + \lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1} = 1 + \lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1} \cdot \frac{\sqrt{x} + 1}{\sqrt{x} + 1}
\]

\[
= 1 + \lim_{x \to 1} \frac{x - 1}{x - 1} \cdot \frac{1}{\sqrt{x} + 1} = 1 + \lim_{x \to 1} \frac{1}{\sqrt{x} + 1} = 1 + \frac{1}{2} = \frac{3}{2}
\]