1. Solve the following matrix equation for $x$ and $y$:

$$2 \begin{bmatrix} 3 & x \\ y & 4 \end{bmatrix} - \begin{bmatrix} 6 & 4 \\ -1 & 2 \end{bmatrix}^T = \begin{bmatrix} 0 & -3 \\ 10 & 6 \end{bmatrix}$$

The value of $2y + x$ is

A) $-1.5$  
B) 5.5  
C) 6.5  
D) 12  
E) none of these

**Solution:** If you expand the matrices, do the transpose, and equate coefficients, you get

$$6 - 6 = 0$$
$$2x - (-1) = -3$$
$$2y - 4 = 10$$
$$8 - 2 = 6$$

This can be solved for $x = -2$ and $y = 7$, which gives $2y + x = 12$, so the answer is D.

2. The Tri-City Office Equipment Corporation sells an imported calculator and performs preventive maintenance and repair service on this calculator. The data collected below is from 8 recent calls from users for routine preventive maintenance. For each call, $x$ is the number of machines serviced and $y$ is the number of minutes spent by the service person. For 7 calculators to be serviced, use regression to estimate how long the service call will take.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>17</td>
<td>25</td>
<td>33</td>
<td>49</td>
<td>68</td>
<td>75</td>
<td>96</td>
<td>115</td>
</tr>
</tbody>
</table>

Use regression to estimate how long the service call will take if 7 calculators are to be serviced.

A) 79.56  
B) 75  
C) 95.05  
D) 80.2  
E) none of these

**Solution:** Using linear regression on the TI-83, we have: \{1, 2, 2, 4, 5, 7, 9, 10\} $\rightarrow L_1$ and \{17, 25, 33, 49, 68, 75, 96, 115\} $\rightarrow L_2$. This gives $y = ax + b$ where $a = 10.225$ and $b = 8.625$. When $x = 7$, this gives $y = 80.2$ so the correct answer is D.

3. The weekly demand equation for Krispy Kritters cereal is given by $D(x) = -0.005x + 3$. The supplier of Krispy Kritters cereal will not market Krispy Kritters if the price per box is $1.50 or less. If the price is $2.00 per box, they will make 200 boxes available per week. What is the equilibrium quantity for Krispy Kritters cereal?

A) no equilibrium quantity  
B) 2.5 boxes  
C) 200 boxes  
D) 400 boxes  
E) none of these

**Solution:** First we need to calculate the supply function. When $x = 0$, $p = $1.50. When $x = 200$ $p = $2.00. These two points determine the supply curve $p = 1.5 + 0.0025x$. Setting the supply and the demand functions to be equal, we get $1.5 + 0.0025x = 3.0 - 0.005x$ this can be solved for $x = 200$. The correct answer is C.

4. The following augmented matrix has how many solutions? The variables are $x$ and $y$.

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

A) exactly one solution  
B) no solution  
C) infinitely many solutions  
D) exactly two solutions  
E) none of these
Solution: Converting this back to a system of 3 equations in 2 unknowns, we see that

\[
\begin{align*}
x &= 4 \\
y &= -3 \\
z &= 0
\end{align*}
\]

which has a unique solution. The correct answer is therefore A.

5. A printing machine has an original value of $100,000 and is to be depreciated linearly over five years. The machine will have a scrap value of $30,000. What is the rate of depreciation of the printing machine?

A) 100,000  B) $-14,000t + 100,000$  C) $-7,000t + 50,000$  D) 14,000  E) none of these

Solution: When \( t = 0 \) the prices is given by $100,000.00. When \( t = 5 \) the prices is given by $30,000.00. The slope is given by $-14,000$. The equation giving price as a function of time is \( p = 100,000 - 14,000t \). The correct answer is B. The response D would not be correct since it is positive...

6. Which of the following inequalities is false for the shaded region shown below.

![Graph](image)

d) \( x + y \leq 1 \)
b) \( x + y \geq 1 \)
a) \( x \leq 1 \)
c) \( y \geq 0 \)
e) none of the above

Solution: Checking each inequality by picking a point in the region (for example the point \((0.75, 0.75)\)), we see that the inequality d) is not true. Therefore d) is the correct answer.

7. For which value of \( k \) does the following system not have a solution:

\[
\begin{align*}
x + 2y &= 1 \\
3x + ky &= 0
\end{align*}
\]

A) $-1$  B) 6  C) 0  D) 1  E) none of these

Solution: The two equations will not have a solution if the lines are parallel. The slope of the first is $-1/2$. The slope of the second is $-3/k$. If these are equal, then $k = 6$. The correct answer is B).
8. Find the equation of the line that passes through the points (1, 2) and (5, 5).

A) \(3x/4 + 5\)  \hspace{1cm} B) \(3x + 5\)  \hspace{1cm} C) \((3x + 5)/4\)  \hspace{1cm} D) \(3x/4 - 3/4\)  \hspace{1cm} E) none of these

**Solution:** We can use the formula
\[
\frac{y - 2}{x - 1} = \frac{5 - 2}{5 - 1} = \frac{3}{4}
\]
when can be solved for
\[
y = 2 + \frac{3}{4}(x - 1) = \frac{3}{4}x + \frac{5}{4} = \frac{(3x + 5)}{4}
\]
The correct answer is C).

9. Find the value of \(a\) such that the line passing through the points \((a, 1)\) and \((5, 4)\) is parallel to the line passing through the points \((a, 1)\) and \((0, 4)\).

A) 0  \hspace{1cm} B) 1  \hspace{1cm} C) 5/6  \hspace{1cm} D) 6/5  \hspace{1cm} E) no such value

**Solution:** The slope of the first line is given by \(m = \frac{4 - a}{5 - a}\). The slope of the second is given by \(m = \frac{4 - 1}{5 - a}\). Equating these two leads to \(-a = 5 - a\) which has no solution. Therefore the correct answer is E).

10. For which value of \(K\) is the line given by \(3x + Ky = 5\) perpendicular to the line given by \(2x - 3y = 6\).

A) 2  \hspace{1cm} B) 0  \hspace{1cm} C) 1  \hspace{1cm} D) -2  \hspace{1cm} E) none of these

**Solution:** The slope of the first line is given by \(m = \frac{2}{3}\). The slope of the second is given by \(\frac{2}{3}\). The perpendicular to the second line has slope \(-\frac{3}{2}\). Equating expressions leads to \(K = 2\) so answer A) is correct.

11. A school district is concerned about the Math SAT scores of its graduating seniors. After 5 years of an intensive on “fundamentals” the ISD gathers the following results:

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAT</td>
<td>506</td>
<td>512</td>
<td>508</td>
<td>509</td>
<td>511</td>
</tr>
</tbody>
</table>

What will be the predicted Math SAT score in the 10th year of this program.

**Solution:** Using linear regression on the TI-83 leads to \(y = ax + b\) where \(a = 0.7\) and \(b = 507.1\). When \(x = 10\), \(y = 514.1\). This should be rounded to 514.

12. A certain software company (located in the Northwest) manufactures an OS for the PC. The fixed cost for development is $10M (10 million dollars). The real manufacturing cost (including the CD, packaging and documentation) is $7.95 per item. The retail price of the software is $99.95. Find the break-even point.

**Solution:** The cost function is given by \(C(x) = 10,000,000 + 7.95x\). The revenue function is given by \(R(x) = 99.95x\). At the break even point, \(R = C\) (\(P = 0\)) so we must solve
\[
10,000,000 + 7.95x = 99.95x
\]
or \(92.0x = 10,000,000\). This has the solution \(x = 108,695.6522\) which should be rounded to 108,696 units.
13. As accurately as possible, graph the region satisfying the inequalities. **Label the coordinates of the vertices.**

\[
\begin{align*}
4x + y & \geq 16 \\
y & \leq 10 \\
x & \leq 8 \\
x + 2y & \geq 10
\end{align*}
\]

**Solution:**

![Graph showing the region satisfying the inequalities with labeled vertices (8,10), (22/7, 24/7), (1.5,10), and (8,1).]

14. Solve the following system of equations, using the matrix inverse method.

\[
\begin{align*}
x_1 + x_2 & - x_3 + x_4 = -1 \\
x_1 & - x_2 + x_3 + 4x_4 = -6 \\
3x_1 + x_2 & - x_3 + 2x_4 = -4 \\
5x_1 + x_2 & - 3x_3 + x_4 = -9
\end{align*}
\]

**Solution:**

a) Write down the inverse matrix. The inverse matrix is given by

\[
\begin{bmatrix}
1 & 1 & -1 & 1 \\
1 & -1 & 1 & 4 \\
3 & 1 & -1 & 2 \\
5 & 1 & -3 & 1
\end{bmatrix}
\]

\[
= \begin{bmatrix}
-3/4 & -1/8 & 5/8 & 0 \\
1/4 & -3/8 & 7/8 & -1/2 \\
-1 & -1/4 & 5/4 & -1/2 \\
1/2 & 1/4 & -1/4 & 0
\end{bmatrix}^{-1}
\]

b) Write down the solution.

Applying the inverse matrix to the right hand side of the equation, or using the rref() function, we obtain \(x_1 = -1, x_2 = 3, x_3 = 2, x_4 = -1\).

15. As Vice President of Marketing for SONY, you are given the following problem: A survey of retailers indicates that the new digital camera will sell 10,000 units a month at a price of $800.00 If the price is reduced to $700.00 they project that 20,000 units a month will be sold. At a cost of $600.00 the factories will produce 5,000 units, but if the cost increases to $900.00 the factories will produce 50,000 units.

**Solution:**

a) Compute the Supply function.

We have two pieces of information. When \(p = 600\) then \(x = 5000\) and when \(p = 900\) then \(x = 50000\). This leads to the supply equation \(p = 1/150x + 5662/3\).

b) Compute the Demand function.

We also have two pieces of information. When \(p = 800\) then \(x = 10000\) and when \(p = 700\) then \(x = 20000\). This leads to the demand equation \(p = -1/100x + 900\).

c) Find the market equilibrium (price and quantity). At equilibrium, we equate the supply and demand equations which leads to \(x = 20000\) and \(p = 700\).