1. Assume the probability of a boy being born is the same as a girl. The probability that in a family of 5 children, three or more children will be girls is given by

A) \( C(5, 3) \)  
B) \( C(5, 3)(.5)^3 \)  
C) \( [C(5, 3) + C(5, 4) + C(5, 5)]/5 \)  
D) \( [C(5, 3) + C(5, 4) + C(5, 5)]/32 \)  
E) none of these

**Solution:** The probability of a girl being born is 0.5. The probability of exactly 3 girls and 2 boys being born is \((0.5)^3(0.5)^2\) and the number of ways this can happen is \(C(5,3)\). Therefore the probability of exactly 3 girls and 2 boys being born is \(C(5,3)(0.5)^5\). The probability of exactly 4 girls and 1 boy is \(C(5,4)(0.5)^5\) and the probability of exactly 5 girls is \(C(5,5)(0.5)^5\). The probability of 3 or more girls is the sum: \([C(5,3) + C(5,4) + C(5,5)](0.5)^5\). Since \((0.5)^5 = (1/2)^5 = 1/32\) the correct answer is C.

2. A raffle is being given by a local church. There will be one first prize awarded, two second prizes (of $75.00), 5 third prizes (of $50.00), and 10 fourth prizes (of $25.00). 1000 tickets will be sold for 1 dollar each. How much must the first prize be in order for the raffle to be fair?

A) 100  
B) 200  
C) 150  
D) 350  
E) none of these

**Solution:** Let \(X\) be the amount of the first prize. If the game is “fair” the expected value of winnings is zero:

\[
E(x) = \frac{1}{1000}(X - 1) + \frac{2}{1000}(75 - 1) + \frac{5}{1000}(50 - 1) + \frac{10}{1000}(25 - 1) + \frac{982}{1000}(0 - 1) = 0
\]

Multiplying both sides by 1000, we have

\[
x + 2 \times 75 + 5 \times 50 + 10 \times 25 - (1 + 2 + 5 + 10 + 982) = 0
\]

or \(x + 650 = 1000\). Therefore \(x = 350\) and the correct answer is D.

Another way of arriving at the same answer is to note that in a fair game, the sum of the prizes must add up to the ticket revenue.

3. A buyer for a local department store is considering buying a batch of clothing for $50,000.00 She estimates that the store will be able to sell the clothing for $70,000.00 with probability 0.30, $65,000.00 with probability 0.50 and $60,000.00 with probability 0.20. If she gets a 10% commission (based on profit) on the sales of the clothing, she can expect to receive

A) 1,050.00  
B) 1,550.00  
C) 750.00  
D) 6,550.00  
E) none of these

**Solution:** The expected value for the sales is

\[
E(X) = 0.3 \times 70,000 + 0.5 \times 65,000 + 0.2 \times 60,000 = 65,500
\]

The resulting profit is 65,500 - 50,000 = 15,500 and her commission would be 0.1 * 15,500 = 1,550. The correct answer is B.

4. Given \(P(A) = 0.2, P(B) = 0.5\) and \(P(A \cup B) = 0.6\) which of the following is correct

A) A and B are independent  
B) A and B are mutually exclusive  
C) A and B are dependent  
D) A and B are complementary  
E) none of these

**Solution:** Since \(P(A \cup B) = P(A) + P(B) - P(A \cap B)\), 0.6 = 0.2 + 0.5 - \(P(A \cap B)\) which gives \(P(A \cap B) = 0.1\). Since \(P(A) \times P(B) = 0.2 \times 0.5 = 0.1\) \(P(A \cap B) = P(A) \times P(B)\) which means that A and B are independent. The correct answer is A.
5. If \( P(A|B) = P(B|A) \) and \( P(A \cap B) \neq 0 \) then \( P(A) = P(B) \)

**Solution:** If \( P(A|B) = P(B|A) \) then

\[
\frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)}
\]

Cross-multiplying, \( P(A) \cdot P(A \cap B) = P(B) \cdot P(A \cap B) \). Since \( P(A \cap B) \neq 0 \), \( P(A) = P(B) \). The answer is T.

(A) True  (B) False

6. A random variable \( X \) has mean \( \mu = 10.0 \) and standard deviation \( \sigma = 1.5 \). The probability that \( P(8 \leq X \leq 12) \) is greater than

A) \( \frac{4}{3} \)  
B) \( \frac{7}{16} \)  
C) \( \frac{9}{16} \)  
D) \( \frac{2}{3} \)  
E) none of these

**Solution:** We use the Chebycheff inequality. First we find the value of \( k \) by setting \( \mu + k\sigma = 10 + 1.5k = 12 \) which means that \( k = \frac{2}{1.5} = \frac{4}{3} \). The probability that \( P(8 \leq X \leq 12) \) is therefore greater than

\[
1 - \frac{1}{k^2} = 1 - \frac{9}{16} = \frac{7}{16}
\]

The correct answer is B.

7. A Bernoulli distribution with 100 objects, and a probability of success equal to 0.2 can be replaced with a normal distribution with

A) \( \mu = 20, \sigma = 16 \)  
B) \( \mu = 16, \sigma = 20 \)  
C) \( \mu = 20, \sigma = 4 \)  
D) \( \mu = 80, \sigma = 4 \)  
E) none of these

**Solution:** With \( N = 100, p = 0.2 \) and \( q = 0.8 \) we have \( \mu = Np = 100 \cdot 0.2 = 20 \) and \( \sigma = \sqrt{Npq} = \sqrt{100 \cdot 0.2 \cdot 0.8} = \sqrt{16} = 4 \). The correct answer is C.

8. Suppose \( X \) is a normally distributed random variable, with \( \mu = 10 \) and \( \sigma = 2 \). Let \( Z \) be a random variable with a standard normal distribution, then \( P(8 < X < 11) \) is the same as

A) \( P(-1 < Z < 0.5) \)  
B) \( P(-1 < Z < 1) \)  
C) \( P(0.5 < Z < 1.0) \)  
D) \( P(-2 < Z < 1.0) \)  
E) none of these

**Solution:** The standard normal variable is given by \( Z = \frac{X - \mu}{\sigma} = \frac{X - 10}{2} \).

\[
8 < X < 11
\]

\[
-2 = 8 - 10 < X - 10 < 11 - 10 = 1
\]

\[
-1 = \frac{-2}{2} < \frac{X - 10}{2} = \frac{1}{2}
\]

\[
-1 < Z < \frac{1}{2}
\]

The correct answer is A.
9. If 1% of the eggs in a large shipment are broken, what is the probability that a customer checks 3 cartons before finding a fourth carton with no broken eggs.

Solution: The probability that an egg will be broken is given by 0.01. The probability that an egg will be unbroken is 0.99. The probability that a carton of 12 eggs will be unbroken is \((0.99)^{12} = 0.8864\). The probability that one or more eggs will be broken in the carton is \(1 - (0.99)^{12} = 0.1136\). The probability that one will get 3 cartons (with one or broken eggs) and then one unbroken carton is \((0.1136)^3(0.8864) = 0.001299\). The answer is 0.13%.

10. From solving the birthday problem in class, we know that the probability of 2 or more people having the same birthday in a group of 25 people is approximately 0.569

Find the probability that exactly two people have the same birthday, in a group of 25 people. (Assume 365 days in a year)

Solution: There are \(C(25, 2)\) ways for exactly 2 people to have the same birthday, and the rest different birthdays. There are 365 birthdays for the 2 people to share, and \(364\)...342\) combinations of birthdays for the remaining 23 people. The probability is therefore

\[
\frac{C(25, 2) \cdot 365 \cdot 364 \cdot ... \cdot 342}{365^{25}} = C(25, 2) \cdot \frac{365 \cdot 364 \cdot ... \cdot 342}{365^{25}} = C(25, 2) \cdot \frac{P(365, 24)}{365^{25}} = 0.37944
\]

The answer is 37.944%. This is reasonable, compared to 56.9%.

11. Given the following table

<table>
<thead>
<tr>
<th>Drivers</th>
<th>% Drivers in Group</th>
<th>% Stopped for Moving Violation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group I (using seat belts)</td>
<td>64.0%</td>
<td>0.2%</td>
</tr>
<tr>
<td>Group II (not using seat belts)</td>
<td>36.0%</td>
<td>0.5%</td>
</tr>
</tbody>
</table>

If a driver is stopped for a moving violation, what is the probability that

a) He or she will have a seat belt on?

b) He or she will not have a seat belt on?

Solution: By looking at the tree diagram below, with S being the event of wearing seatbelts, and V the event of being ticketed with a moving violation:
The answer to a) is
\[ P(S|V) = \frac{(0.64)(0.002)}{(0.64)(0.002) + (0.36)(0.005)} = 0.41558 \]

The answer to b) is
\[ P(S^c|V) = \frac{(0.36)(0.005)}{(0.64)(0.002) + (0.36)(0.005)} = 0.58441 \]

Note, these must sum to 1.0!

12. Given the following tree diagram

![Tree Diagram](image)

Find the probability \( P(B|D^c) \).

**Solution:** By a straightforward application of Bayes’ Theorem,
\[ P(B|D^c) = \frac{(0.4)(0.7)}{(0.2)(0.8) + (0.4)(0.7) + (0.4)(0.5)} = 0.4375 \]

13. An experiment consists of throwing two standard 6 sided dice on the table, and then taking the product of the numbers thrown. If \( X \) is the random variable denoting the product of the die

The table of the possible values for \( X \) is given below:

<table>
<thead>
<tr>
<th>( X )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
</tr>
</tbody>
</table>

The frequency distribution is
\[ \{11, 22, 32, 43, 64, 52, 82, 91, 102, 124, 152, 161, 182, 202, 242, 251, 302, 361 \} \]

a) Graph the probability distribution (histogram) associated with \( X \).
b) What is \( E(X) \)

\[
E(X) = \frac{1}{36} + \frac{2}{36} + \frac{2}{36} + \frac{3}{36} + \frac{2}{36} + \frac{4}{36} + \frac{2}{36} + \frac{5}{36} + \frac{2}{36} + \frac{6}{36} + \frac{2}{36} + \frac{8}{36} + \frac{2}{36} + \frac{9}{36} + \frac{1}{36} + \frac{10}{36} + \frac{2}{36} + \frac{12}{36} + \frac{4}{36} + \frac{15}{36} + \frac{2}{36} + \frac{16}{36} + \frac{1}{36} + \frac{18}{36} + \frac{2}{36} + \frac{20}{36} + \frac{2}{36} + \frac{24}{36} + \frac{2}{36} + \frac{25}{36} + \frac{1}{36} + \frac{30}{36} - \frac{36}{36} = 12.25
\]

14. An urn contains 5 red balls, 6 white balls, and 4 blue balls. What is the probability that 2 red balls and 2 white balls will be drawn in the first 5 balls.

**Solution**: Note, there are three possibilities: 2-red, 2-white, 1-blue; 3-red, 2-white; 2-red, 3-white. There are \( C(15, 5) \) ways to drawing 5 balls from 15.

The probability that one gets 2-red, 2-white and 1-blue is

\[
\frac{C(5, 2) \cdot C(6, 2) \cdot C(4, 1)}{C(15, 5)}
\]

The probability that one gets 3-red and 2-white is

\[
\frac{C(5, 3) \cdot C(6, 2) \cdot C(4, 0)}{C(15, 5)}
\]

The probability that one gets 2-red and 3-white is

\[
\frac{C(5, 2) \cdot C(6, 2) \cdot C(4, 0)}{C(15, 5)}
\]

The answer is the sum of these probabilities:

\[
\frac{C(5, 2) \cdot C(6, 2) \cdot C(4, 1)}{C(15, 5)} + \frac{C(5, 3) \cdot C(6, 2) \cdot C(4, 0)}{C(15, 5)} + \frac{C(5, 2) \cdot C(6, 2) \cdot C(4, 0)}{C(15, 5)}
\]