Math 304 Homework 2.2

> with(linalg):

1 Problem 1

1.1 Problem 1a

expand about 1st column or 1st row

> det([[0,0,3],[0,4,1],[2,3,1]]);
-24

1.2 Problem 1b

expand about 1st column, or 3rd row

> det([[1,1,1,3],[0,3,1,1],[0,0,2,2],[-1,-1,-1,2]]);
30

1.3 Problem 1c

expand about 1st column, note that minor is 3x3 identity!

> det([[0,0,0,1],[1,0,0,0],[0,1,0,0],[0,0,1,0]]);
-1

2 Problem 2

> A := array(1..4,1..4,
> [[0,1,2,3],[1,1,1,1],[-2,-2,3,3],[1,2,-2,-3]]);

\[
A := \begin{bmatrix}
0 & 1 & 2 & 3 \\
1 & 1 & 1 & 1 \\
-2 & -2 & 3 & 3 \\
1 & 2 & -2 & -3
\end{bmatrix}
\]

2.1 Problem 2a

row reduce to triangular form

> gausselim(A);

\[
\begin{bmatrix}
1 & 1 & 1 & 1 \\
0 & 1 & 2 & 3 \\
0 & 0 & 5 & 5 \\
0 & 0 & 0 & -2
\end{bmatrix}
\]

determinant is the product of diagonal elements (to within a +/- sign!)

> det(A);
2.2 Problem 2b

The first determinant is arrived by i) interchanging 2nd and 3rd row, then ii) interchanging 3rd and 4th row

This means determinant is \((-1)(-1)\)det(A)=10

\[ \text{det}([0,1,2,3],[-2,-2,3,3],[1,2,-2,-3],[1,1,1,1]) ; \]

The second determinant is formed by
i) adding 2nd row to 3rd row, then
ii) adding 2nd row to 4th row

This leaves the determinant unchanged

\[ \text{det}([[0,1,2,3],[1,1,1,1],[-1,-1,4,4],[2,3,-1,-2]]) ; \]

The sum of these determinants is 10+20=20

3 Problem 3

3.1 Problem 3a

\[ \text{det}([[3,1],[6,2]]) ; \]

0

Matrix is singular!

3.2 Problem 3b

\[ \text{det}([[3,1],[4,2]]) ; \]

2

Matrix is non-singular!

3.3 Problem 3c

\[ \text{det}([[3,3,1],[0,1,2],[0,2,3]]) ; \]

-3

Matrix is non-singular!

3.4 Problem 3d

\[ \text{det}([[2,1,1],[4,3,5],[2,1,2]]) ; \]

2

Matrix is non-singular!

3.5 Problem 3e

\[ \text{det}([[2,-1,3],[-1,2,-2],[1,4,0]]) ; \]

0

Matrix is singular!
3.6 Problem 3f
> det([[1,1,1,1],[2,-1,3,2],[0,1,2,1],[0,0,7,3]]);
0
Matrix is singular!

4 Problem 4
> A := array([[1,1,1,1],[1,9,c],[1,c,3]]);
A :=

> det(A);
15 - c^2 + 2c
> solve(det(A)=0,c);
-3, 5

5 Problem 6
Since 1 = det(I) = det(A*A^(-1)) we can solve for det(A^(-1)) by det(A^(-1)) = 1/det(A)

6 Problem 7
6.1 Problem 7a
det(AB) = det(A)*det(B) = 4*5 = 20

6.2 Problem 7b
det(3A) = 3^3 * det(A) = 27*4 = 108

6.3 Problem 7c
det(2AB) = 2^3 * det(A) * det(B) = 8*4*5 = 160

6.4 Problem 7d
det(A^-1 B) = det(A^-1) det(B) = 1/det(A) * det(B) = 1/4 * 5 = 5/4

7 Problem 10
> V := array(1..3,1..3,
> [[1,x1,x1^2],
> [1,x2,x2^2],
> [1,x3,x3^3]]);
\[
V := \begin{bmatrix}
1 & x1 & x1^2 \\
1 & x2 & x2^2 \\
1 & x3 & x3^3 \\
\end{bmatrix}
\]

\[
\text{det}(V) = x_2 x_3^3 - x_2^2 x_3 - x_1 x_3^3 + x_1^2 x_3 + x_1 x_2^2 - x_1^2 x_2
\]

\[
\text{det}(V) = (x_2 - x_1)(x_3 - x_1)(x_3 - x_2)
\]

\[
\text{det}(V) = x_2 x_3^2 - x_2^2 x_3 + x_1 x_2^2 - x_1 x_3^2 + x_1^2 x_3 - x_1^2 x_2
\]

determinant vanishes if \(x_1=x_2\), \(x_2=x_3\), or \(x_1=x_3\). \(V\) is non-singular if and only if \(x_1, x_2, x_3\) are distinct!