Greek Construction Problems

I. Given a straight line, and a unit circle, it is clear that one can construct a unit length line segment.

II. Given the unit segment, one can extend it to an integer multiple (which gives us multiplication of integer lengths). Clearly, we can also subtract integer multiples of lengths.

III. We can also perform division by using similar triangles
By similar triangles it is clear that

\[ \frac{y}{x} = \frac{z}{1} \]

IV. Finally square roots can be obtained by using the pythagorean theorem. Unit lengths on the sides gives us the length \( \sqrt{2} \)

Therefore, using a straightedge and a unit circle (compass) we can construct the set of all rational lengths, and their square roots.

**Question 1:** Is this enough to compute \( \pi \)?

**Question 2:** Is this enough to compute \( \sqrt{2} \)?

**Question 3:** Can you solve an arbitrary quadratic equation with integer coefficients?

**Question 4:** Can you solve an arbitrary cubic equation with integer coefficients?

**Question 5:** What implications does this have for the 3 classical greek construction problems: Squaring the circle, doubling the cube, and trisecting the angle?

**Note** You may find it helpful to consult Chapter 6 of Boas & Geller.