Problem 1 (20pts). Show that each the distance postulates $D1, D2$ and $D3$ is a consequence of the postulate $D4$.

Problem 2 (20pts). Let $A, B, C, D$ be points in space. Show that if $A-B-C$ and $B-C-D$, then $A-B-D$ and $A-C-D$.

Problem 3 (20pts). Let $A, B, C, D$ be points in space. Show that if $A-B-C$ and $A-D-C$, then either $A-B-D-C$, or $A-D-B-C$, or $B = D$.

Problem 4 (20pts). Given four spherical beads of different colors, in how many essentially different ways is it possible to arrange them in a string so as to make a four-bead necklace? (The string is so thin that the knot can slip through the holes in the beads. This is a problem about betweeness on a circle, and the answer indicates that the idea of betweeness on a circle is more peculiar than one might have supposed.

Problem 5 (20pts). Show that given a ray $\overrightarrow{AB}$, there is a coordinate system $f$ on the line $\overrightarrow{AB}$ such that $\overrightarrow{AB} = \{P \mid f(P) \geq 0\}$.

Problem 6 (20pts). Let $A$ and $B$ be two points in space, and let $D, E, F$ be three non-collinear points. If the line $\overrightarrow{AB}$ contains only one of the points $D, E, F$, prove that each of the lines $\overrightarrow{DE}, \overrightarrow{DF}$ and $\overrightarrow{EF}$ intersects $\overrightarrow{AB}$ in at most one point.

Problem 7 (20pts). Prove the following. If $\triangle ABC = \triangle DEF$, then each side of $\triangle ABC$ contains two of the points $D, E$ and $F$.

Problem 8 (20pts). Show that $A$ is not between any two points of the triangle $\triangle ABC$.

Problem 9 (20pts). Prove Theorem 5 (p. 66): If $\triangle ABC = \triangle DEF$, then the points $A, B, C$ are the same as points $C, D, E$ in some order. In other words, $\{A, B, C\} = \{D, E, F\}$.