Problem 1. a Show that a product of a paracompact space with a compact space is paracompact. Use facts from the previous assignment to conclude that $S_\Omega$ is not paracompact.

b Show that $S_\Omega \times [0,1)$ with the lexicographic order topology is not paracompact.

c Is every locally compact Hausdorff space paracompact?


Problem 3. Recall that a collection $\{A_\lambda\}_{\lambda \in A}$ of subsets of a topological space $X$ is called locally finite if every point in $X$ has an open neighborhood which has non-trivial intersection with at most finitely many elements in this family.

a Show that if $\{A_\lambda\}_{\lambda \in A}$ is a locally finite family of subsets of $X$, then so is $\{\overline{A_\lambda}\}_{\lambda \in A}$.

b Let $\{V_i\}_{i \in \mathbb{N}}$ be a countable open cover of a space $X$, and for each $i \in \mathbb{N}$ let $U_i$ be an open set such that $V_i \subset \overline{V_i} \subset U_i$, for all $i \in \mathbb{N}$. Define $W_i = U_i - \bigcup_{k<i} \overline{V_k}$. Show that $\{W_i\}_{i \in \mathbb{N}}$ is a locally finite, open cover of $X$.