Spring 2005 Math 152
Night Before Drill
courtesy: Amy Austin
(covering sections 8.4-9.6)

Note: The second common exam covers 8.8-9.6 only. I placed these problems from 8.4 here for review purposes only.

Section 8.4
1. \[ \int_{2}^{3} \frac{dx}{(x-1)(x+2)} \]
2. \[ \int \frac{dx}{x^2(x-1)} \]
3. \[ \int \frac{5x^2 - 3x + 4}{(x-1)(x^2 + 1)} \] dx
4. \[ \int \frac{x^3}{x^2 - x} \] dx

Section 8.8
5. Find the Midpoint Rule approximation \( M_4 \) to \( \int_{1}^{2} e^{x^2} \) dx. Construct approximating rectangles.
6. Given the table below, use the Trapezoid rule with \( n = 5 \) to approximate \( \int_{1}^{2} f(x) \) dx.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>4.8</td>
<td>5.4</td>
<td>5.8</td>
<td>6.2</td>
<td>6.8</td>
<td>7.0</td>
</tr>
</tbody>
</table>
7. a.) Use Simpson’s rule with \( n = 4 \) to approximate \( \int_{1}^{2} \ln x \) dx.
b.) Use the error bound formula to find an upper bound on the error in using \( S_4 \) to approximate \( \int_{1}^{2} \ln x \) dx.
c.) How large do we need to choose \( n \) so that the approximation \( S_n \) to the integral \( \int_{1}^{2} \ln x \) dx has error less than \( 10^{-5} \)?

Section 8.9
8. Determine whether the following integrals converge or diverge. Evaluate those that converge.

9. For each of the following integrals, determine whether the integral converges or diverges using the comparison theorem.

(a) \( \int_{0}^{\infty} e^{-2x} \) dx
(b) \( \int_{-2}^{2} \frac{1}{x^2} \) dx
(c) \( \int_{0}^{1} \frac{x}{(x^2 - 1)^2} \) dx

Sections 9.1 and 9.2
10. \( xy' - 2y = 2x^4 \), \( y(1) = 0 \)
11. \( y \frac{dy}{dx} = \frac{x^2}{e^{y^2}} \)
12. \( y' = \frac{e^{x-y}}{1+e^x} \), \( y(1) = 0 \)
13. \( y' + 2xy = xe^{-x^2} \)
14. Given that \( \frac{dt}{dx} = \frac{2x}{t^2} \) and \( t(3) = 3 \), find \( t(1) \).
15. A curve passes through the point \((7, 4)\) and has the property that the slope of the curve at every point \( P \) is 5 times the \( y \)-coordinate of \( P \). What is the equation of the curve?
16. A tank contains 1000 liters of brine with 15 kg of dissolved salt. Pure water enters the tank at a rate of 10 liters per minute. The solution is kept mixed and exits the tank at the same rate. How much salt is in the tank after 25 minutes?

Section 9.3
17. Find the length of the curve \( x = \sin(2t), y = \cos(2t), 0 \leq t \leq \frac{\pi}{4} \)
18. Find the length of the curve \( x = \frac{2}{3}y^{3/2}, 0 \leq y \leq 4 \).
Section 9.4

19. Find the surface area obtained by rotating the curve \( y = x^2, \ 0 \leq x \leq 2 \) about the y axis.

20. Find the integral that gives surface area obtained by rotating the curve \( y = x^4, \ 0 \leq x \leq 1 \) about the x axis. Do not evaluate the integral. Do the same for rotation around the y axis.

21. The curve \( y = \cos x, \ 0 \leq x \leq \frac{\pi}{2} \) is rotated around the x axis. Set up both a \( dx \) and a \( dy \) integral that gives the resulting surface area. Do not evaluate either integral.

22. Find the surface area obtained by rotating the curve \( x = 3 \sin t, \ y = 3 \cos t, \ 0 \leq t \leq \frac{\pi}{4} \) around the y axis.

Section 9.5

Note: In this section, you will only be held responsible for the \( x \)-coordinate of the centroid.

23. Find the center of mass of the system consisting of the masses 2 g, 5 g, 3 g, and 2 g located at the points \( x = -10, \ x = 1, \ x = 2, \) and \( x = 6 \) along the x axis, respectively.

24. Find the moment about the x axis, the moment about the y-axis, and the centroid of the system consisting of the masses 1 g, 3 g and 7 g located at the points \((-2,3), \ (5,1) \) and \((6,-3)\), respectively.

25. Let \( R \) be the region bounded by bounded by \( y = x^4 - 1, \ y = 0, \ x = 1, \ x = 2 \). Assume the uniform density of \( R \) is \( \rho = 9 \). Find the moment of the system about the y-axis and the moment of the system about the x-axis. Also, find the centroid of \( R \).

26. Find the centroid of the region bounded by \( y = \cos x, \ y = \sin x, \ 0 \leq x \leq \frac{\pi}{4} \).

Section 9.6

27. A rectangular pool 20 meters long, 15 meters wide and 4 meters deep is filled with a fluid of density 1020 kg/m\(^3\) to a depth of 3 meters.

a.) Find the hydrostatic force on the bottom of the pool.

b.) Find the hydrostatic force on the end of the pool.

28. A tank contains water. The ends of the tank are vertical and have the shape described below. Find the hydrostatic force against the end of the tank.

a.) Semicircle of diameter 6 feet, with the diameter at the top. Assume the tank is full. The weight density of water is 62.5 pounds per cubic foot.

b.) Isosceles triangle with height 4 feet, base length 2 feet with vertex at the top. The water level is 3 feet deep, measuring from the bottom. The weight density of water is 62.5 pounds per cubic foot.

c.) The region bounded by \( y = x^2, \ y = 4 \).

29. A 2x2x2 m\(^3\) cube is resting flat on its side at the bottom of a tank full of water to a depth of 20 m. The density of water is 1000 kg/m\(^3\).

a.) Find the force of water on the top of the cube.

b.) Find the force of water on any one of the four vertical exposed sides.