Section 10.1

1. Determine whether the following sequences converge or diverge. If the sequence converges, find the limit. If the sequence diverges, explain why.

a.) $a_n = \frac{5 \cos n}{n}$

b.) $a_n = 3 + \frac{(-1)^n}{n}$

c.) $a_n = \arccos \left( \frac{(n+1)!}{(n+3)!} \right)$

d.) $a_n = \frac{(\arctan n)^5}{n^2}$

Section 10.2

2. Find the sum of the following series.

a.) $\sum_{n=1}^{\infty} \left( \sin \frac{1}{n} - \sin \frac{1}{n+2} \right)$

b.) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

c.) $\sum_{n=1}^{\infty} 5 \left( \frac{2}{7} \right)^n$

d.) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} + 3^n}{4^n}$

e.) $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{5^{n+1}}$

f.) $\frac{5}{2} - \frac{5}{4} + \frac{5}{8} - \frac{5}{16} - ... + ...$

3. If the $n$th partial sum of the series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{n-1}{n+1}$, find $a_n$ and the sum of the series $\sum_{n=1}^{\infty} a_n$.

Section 10.3 and 10.4

4. Determine whether the following series converge or diverge. You must name the test, and apply the test completely and correctly.

a.) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

b.) $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n+1}$

c.) $\sum_{n=1}^{\infty} \frac{(n+1)(-3)^n}{\sqrt{n^4 n}}$

d.) $\sum_{n=2}^{\infty} \frac{(-10)^{n-1}n^2}{(2n-3)!}$

e.) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2 + 1}$

5. Determine whether the following series converge or diverge. For those that converge, determine whether they are absolutely convergent.

a.) $\sum_{n=2}^{\infty} \frac{(-1)^n}{3n - 1}$

b.) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^3 + 1}$

c.) $\sum_{n=1}^{\infty} \frac{2}{n^3}$

d.) $\sum_{n=1}^{\infty} \frac{(-10,000)^n n!}{(2n + 1)!}$

6. Use $S_{10}$ to estimate the sum of the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$. Give an estimate on the remainder.

7. Consider the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^5}$.

a.) Use the first 5 terms to estimate the sum.

b.) Estimate the error in the approximation $s_5$ to the sum of the series.

8. How many terms of the series do we need to add in order to find the sum to the indicated accuracy? $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$, (error < $\frac{1}{100}$).

Section 10.5

9. For the following power series, find the radius and interval of convergence.

a.) $\sum_{n=0}^{\infty} \frac{(\pi x)^n}{n^2 + 2}$
b.) \[ \sum_{n=0}^{\infty} \frac{(2x-1)^n}{n!4^n} \]

c.) \[ \sum_{n=1}^{\infty} \frac{(2n-1)!(x+2)^{n-1}}{5^{n-1}} \]

d.) \[ \sum_{n=1}^{\infty} \frac{(-1)^n(x-1)^n}{\sqrt{n}} \]

10. Suppose it is known that \[ \sum_{n=0}^{\infty} c_n x^n \] is convergent when \( x = 4 \) and divergent when \( x = 6 \). What can be said about the convergence or divergence of the following series:

a.) \[ \sum_{n=0}^{\infty} c_n 8^n \]

b.) \[ \sum_{n=0}^{\infty} c_n (-3)^n \]

c.) \[ \sum_{n=0}^{\infty} c_n (-4)^n \]

d.) \[ \sum_{n=0}^{\infty} c_n (5)^n \]

Section 10.6

11. Find a power series for the following functions and find the corresponding radius of convergence.

a.) \( f(x) = \frac{1}{1-x} \)

b.) \( f(x) = \frac{1}{4 + x^2} \)

c.) \( f(x) = \ln(2-x) \)

d.) \( f(x) = \arctan x \)

e.) \( f(x) = \frac{x}{(x^2 + 1)^2} \)

12. Write \( \int_{0}^{0.5} \frac{1}{1 + x^4} \, dx \) as an infinite series.

Section 10.7

13. Find the Taylor Series for \( f(x) = e^{4x} \) at \( x = -1 \).

14. Find the first three nonzero terms of the Taylor Series for \( f(x) = \frac{1}{x^2} \) centered at \( x = 3 \).

15. Find a Maclaurin series for \( \int \frac{\cos(x^2)}{x} \, dx \)

16. Express \( \int_{0}^{1} e^{-x^2} \, dx \) as an infinite series. Use the first 2 terms of this series to approximate the sum and estimate the error.

17. Find the coefficient of \( (x-4)^3 \) in the Taylor Series for the function \( f(x) = \sqrt{x} \) at \( a = 4 \).

18. Consider \( f(x) = \frac{\sin x - x + \frac{1}{6} x^3}{x^5} \). This function can be expressed as a series, by using the known Maclaurin series for \( \sin x \). Find the first four nonzero terms of this series.

Section 10.9

For the problems 16-17, answer the following questions:

a.) Find \( T_n(x) \) at the given value of \( a \). Also, find the remainder term.

b.) Use Taylor’s Inequality to estimate the accuracy of the approximation \( T_n(x) \) for \( x \) in the given interval.

19. \( f(x) = \frac{2}{x^2}, \ n = 4, \ a = 2, \ 1 \leq x \leq 2.5 \)

20. \( f(x) = \sqrt{1 + x}, \ n = 2, \ a = 1, \ 0 \leq x \leq 1.1 \)