Instructions Please write your name in the upper right-hand corner of the page. Write complete sentences to explain your solutions.

1. Find a linear fractional transformation that takes the triple of points \((0, 1, 2)\) onto \((1, 2, 3)\).

Solution. This transformation is a translation by 1 unit to the right, so you can write down the formula without any calculation: the transformation is \(z \mapsto z + 1\).

2. Find a linear fractional transformation that takes the triple \((1, 0, \infty)\) onto \((\infty, 1, 0)\).

Solution. Suppose the transformation has the general form \(\frac{az + b}{cz + d}\). Since 1 maps to \(\infty\), we must have \(c + d = 0\), or \(c = -d\). Since 0 maps to 1, we have \(b/d = 1\), or \(b = d\). Since \(\infty\) maps to 0, we have \(a/c = 0\), or \(a = 0\). We may take \(d\) equal to 1, in which case \(b = 1\) and \(c = -1\); the transformation is given by \(\frac{1}{-z + 1}\).

3. Find a linear fractional transformation that takes the circle centered at 0 with radius 1 onto the circle centered at 3 with radius 2.

Solution. You can implement this transformation by first dilating by a factor of 2 and then translating 3 units to the right. Thus the transformation is \(z \mapsto 2z + 3\).