1. (a) i. Define what it means for a sequence of real numbers to be monotone.
   ii. Give an example of a monotone sequence.
   iii. Give an example of a sequence that is not monotone.

   (b) i. Define what it means for a sequence of real numbers to be a Cauchy sequence.
   ii. Give an example of a Cauchy sequence.
   iii. Give an example of a sequence that is not a Cauchy sequence.

2. (a) State the Bolzano-Weierstrass theorem.
   (b) State the squeeze theorem about limits of sequences.

3. (a) State the definition of “the limit of the sequence \( \{a_n\} \) is \( L \).”
   (b) Use the definition in part (a) to prove that 0 is the limit of the sequence \( \{1/\sqrt{n^2 + 1}\} \).

4. For each of the following statements, say whether the statement is true or false. To support an answer of “true”, you must give an explanation or cite a theorem or supply a proof; to support an answer of “false”, you must exhibit a counterexample.
   (a) If a sequence \( \{a_n\} \) is unbounded, then the sequence \( \{a_n\} \) has no cluster point.
   (b) If a sequence \( \{a_n\} \) converges, then the related sequence \( \{(-1)^na_n\} \) diverges.

5. Prove that the infinite series \( \sum_{n=1}^{\infty} 1/\sqrt{n^2 + 1} \) diverges.
   (Notice that this problem differs from problem 3(b): that problem concerns a sequence, but this problem concerns a series.)