1. State the following three theorems.

(a) the Bolzano-Weierstrass theorem (about sequences)
(b) the intermediate value theorem (about continuous functions)
(c) the mean-value theorem (about differentiable functions)

2. Define the following three notions.

(a) compact interval
(b) Cauchy sequence
(c) Riemann sum

3. (a) State the definition of “the sequence \( \{a_n\}_{n=1}^{\infty} \) converges to the limit \( L \)” in the form “for every \( \epsilon > 0 \ldots \).”

(b) Prove from the definition that the sequence \( \left\{ \frac{1}{2^n} \right\}_{n=1}^{\infty} \) converges to the limit 0.

4. (a) State the definition of “the series \( \sum_{n=1}^{\infty} a_n \) converges to the sum \( S \).”

(b) Prove from the definition that the series \( \sum_{n=1}^{\infty} \frac{1}{2^n} \) converges to the sum 1.

5. (a) State the definition of “the function \( f \) is continuous at the point \( x_0 \)” in the form “for every \( \epsilon > 0 \ldots \).”

(b) Prove from the definition that the function \( f(x) = x^2 \) is continuous at every point \( x_0 \).
6. (a) State the definition of “the function $f$ is differentiable at the point $a$”.
(b) Prove from the definition that the function $f(x) = x^2$ is differentiable at every point $a$.

7. (a) State the definition of “the function $f$ is integrable (in the sense of Riemann) on the interval $[a, b]$” in terms of upper sums and lower sums.
(b) Prove from the definition that the function $f(x) = x^2$ is integrable on the interval $[0, 1]$.

8. This question concerns the power series $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$. In answering this question, you may cite relevant theorems from the course.
(a) Show that the series converges for every $x$ in the closed interval $[-1, 1]$.
(b) Does the series represent an integrable function on the closed interval $[-1, 1]$? Explain why or why not.