Reminder

Second examination is Monday, November 1.

The exam will have a similar format to the format of the first exam.

The exam covers material through section 13.4.

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**Continuity and boundedness**

A continuous function on an interval need not be bounded.
Examples: $1/x$ on the bounded interval $(0, 1)$; $x^2$ on the closed interval $[0, \infty)$.

**Theorem:** Every continuous function $f$ on a compact interval $[a, b]$ is bounded.

**Proof** (different from the proof in the book):
Use from Exercise 11.1/5 that

\[ (*) \text{ every continuous function is locally bounded.} \]

Let $S = \{ c : f \text{ is bounded on the interval } [a, c] \}$. By $(*)$, $S$ is not empty. Let $d = \sup S$. By $(*)$, $d \in S$. If $d < b$, then by $(*)$ some points to the right of $d$ are in $S$, which contradicts that $d$ is an upper bound for $S$. Therefore $d = b$, and we are done.
**Continuity and extreme values**

A continuous function on an interval need not attain a maximum value. Examples: $x^2$ on the bounded interval $(0, 1)$; $\arctan(x)$ on the closed interval $[0, \infty)$.

Theorem: Every continuous function $f$ on a compact interval attains a maximum value (and also attains a minimum value).

Proof by contradiction (different from the proof in the book):

By the previous theorem, $f$ is bounded. Let $M = \sup f(x)$. Suppose the supremum is not attained. Then $M - f(x) > 0$ for all $x$, so $\frac{1}{M-f(x)}$ is continuous.

By the previous theorem, this new function has an upper bound, say $N$. Solve $\frac{1}{M-f(x)} \leq N$ to get $f(x) \leq M - \frac{1}{N}$, contradicting that $M = \sup f(x)$.

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**Homework**

- Read sections 13.3 and 13.4, pages 187–190.
- In preparation for the examination, make a list of the main definitions, concepts, and theorems from sections 7.5 through 13.4.