Announcement

TAMU Math Club Meeting

Monday, September 20

7:00pm in Blocker 627

Speakers:
Dr. Philip Yasskin, “Rascal’s Triangle”
Ms. Edith Andrews, Jane Long Middle School HOSTS program

FREE FOOD

More about subsequences and convergence

Main theorems in Chapter 6

• Nested interval theorem
• Bolzano-Weierstrass theorem

Main concepts in Chapter 6

• cluster point
• Cauchy sequence
• supremum
• lim sup
Nested intervals

**Theorem.** If the closed intervals

\[ [a_1, b_1] \supseteq [a_2, b_2] \supseteq \ldots \supseteq [a_n, b_n] \supseteq \ldots \]

are nested, then the intersection \( \bigcap_{n=1}^{\infty} [a_n, b_n] \) is not empty.

Moreover, if \( \text{length}[a_n, b_n] \to 0 \), then there is exactly one point common to all the intervals.

**Examples**

- The nested intervals \([-1 - 1/n, 1 + 1/n] \) have intersection \([-1, 1] \).
- The nested intervals \([1 - 1/n, 1] \) have intersection \(\{1\} \).
- The nested open intervals \((0, 1/n) \) have empty intersection.

Bolzano-Weierstrass theorem

**Theorem.** A bounded sequence of real numbers has convergent subsequences.

Proof: repeated bisection and the nested interval theorem.

**Examples**

- The sequence \( \{\sin n\}_{n=1}^{\infty} \) has convergent subsequences.
- Let \( x_n \) be the right-most digit of the \( n \)th prime number. Then the sequence \( \{x_n\}_{n=1}^{\infty} \) has convergent subsequences.
**Cluster points**

**Definition**

A *cluster point* of a sequence is the limit of a convergent subsequence. (Another name for the same concept is *accumulation point*.)

**Examples**

- The sequence \( \{(-1)^n\}_{n=1}^\infty \) has two cluster points: namely 1 and \(-1\).
- The sequence \( \{n \sin(n\pi/2)\}_{n=1}^\infty \) has one cluster point: namely 0.

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**Homework**

- Read sections 6.1–6.3, pages 78–83.
- Do Exercises 6.2/1 and 6.3/1 on pages 89–90.