General Statement. You should have a good working knowledge of the following topics from Math 151, 152: limits, differentiation rules, derivative as a rate of change, basic curve sketching with calculus, integration rules, the integral as a sum, vectors in 2 and 3 dimensions, lines and planes in 3-space, functions of several variables and their graphs, partial derivatives and gradients, and the tangent plane to a graph of a function of two variables. The following review exercises should help you review some of these important concepts from Math 152.

Integrals.

1. Do the following integrals

\[ \int \frac{x \, dx}{\sqrt{1 + x^2}} \quad \int xe^{2x} \, dx \quad \int_{3}^{5} \frac{dx}{x^2 - 4} \]

2. Determine whether or not the following integrals converge or diverge

\[ \int_{0}^{\infty} \frac{dx}{\sqrt{x^3 + x}} \quad \int_{0}^{1} \frac{dx}{x \sqrt{x}} \]

3. Set up the integral that computes the area between the line \( y = x - 1 \) and the parabola \( y^2 = 2x + 6 \).

4. Set up the integral that computes the volume of the solid with a base given by the unit circle and with cross sections perpendicular to the base given by equilateral triangles.

5. A heavy rope, 50 feet long, weighs \( 0.5 \text{ lb/ft} \) and hangs over the edge of building 120 feet high. How much work is done in pulling the rope to the top of the building?

Vectors, lines and planes in 3-space

6. Find the angle between the vectors \( \langle 1, 2, 2 \rangle \) and \( \langle 3, 4, 0 \rangle \).

7. Find the equation of the line that is perpendicular to the plane \( 2x + y - 3z = 5 \) and passing through the point \( (4, -1, 2) \). Express the answer in both parametric and symmetric form.

8. Find the equation of the plane that passes through the points \( (1, 0, -3), (0, -2, -4) \) and \( (4, 1, 6) \).

9. Find the equation of the tangent line to the curve \( x = t, y = t^2, z = t^3 \) at the point \( (1, 1, 1) \). Express the answer in both parametric and symmetric form.

10. Find the distance from the point \( (2, 8, 5) \) to the plane \( x - 2y - 2z = 1 \).

Functions of several variables and their graphs
11. Graph the following surfaces

\[ z = x^2 - 2y^2 \quad z^2 + 2x^2 + 4y^2 = 16 \quad z^2 - x^2 + y^2 = 4 \]

12. Describe the level curves of the following functions of two variables. (Recall that a level curve of a function \( f(x, y) \) is a set in the plane of the form \( \{(x, y); f(x, y) = k\} \) where \( k \) is a constant).

\[ f(x, y) = x^2 - y^2 \quad f(x, y) = \frac{x^2 + y^2}{x} \]

13. Describe the level surfaces of the following functions of three variables (Recall that a level surface of a function \( f(x, y, z) \) is a set in the 3-space of the form \( \{(x, y, z); f(x, y, z) = k\} \) where \( k \) is a constant).

\[ f(x, y, z) = x^2 - y^2 - z^2 \quad f(x, y, z) = 3x - 2y + 4z \]

Partial Derivatives, gradients and tangent planes

14. Find the partial derivatives of the following functions with respect to \( x \) and \( y \)

\[ f(x, y) = x \cos(x^2 y) \quad f(x, y) = e^{xy} \sqrt{x^2 + y^2} \]

15. Find a unit vector that points in the direction of maximum increase of the given function at the given point (recall that the gradient points in the direction of maximum increase).

\[ f(x, y) = x^2 y^3 \text{ at } (1, 2) \quad f(x, y, z) = x^2 + y^3 + z^4 \text{ at } (-1, 3, 2) \]

16. Find the equation of the tangent plane to the given surface at the given point.

\[ z = -x^2 y^3 \text{ at } (1, 1, -1) \quad x^2 + 2y^2 - z^2 = 8 \text{ at } (1, 2, 1) \]

17. Indicate the possible directions of the gradient vectors at various points on the following grid that represents the level sets (or contour map) of a function of two variables. What missing information regarding the grid would be helpful in order to more accurately determine the gradient vectors?