Math 222 - Exam I

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1. Write the vector

\[
\begin{pmatrix}
-6 \\
10 \\
-19
\end{pmatrix}
\]

as a linear combination of the vectors

\[v_1 = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \quad v_2 = \begin{pmatrix} -2 \\ 6 \\ -2 \end{pmatrix} \quad v_3 = \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix}\]

Is this linear combination unique? (12 points)

2. Find the elementary matrix \( E \) such that \( EA = B \) where

\[
A = \begin{pmatrix}
3 & 6 & -9 \\
2 & 1 & 0 \\
2 & -1 & 4
\end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix}
3 & 6 & -9 \\
2 & 1 & 0 \\
0 & -5 & 10
\end{pmatrix}
\]

(12 points)

3. What is the dimension of the nullspace of the matrix

\[
\begin{pmatrix}
1 & 2 & -1 & 3 & 4 \\
0 & 0 & 1 & -2 & -5 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\]

Justify your answer. (12 points)
4. Use elementary row operations to reduce the following matrix to an upper triangular matrix. In the process, compute the determinant.

\[
\begin{pmatrix}
2 & 1 & -1 \\
1 & 0 & 1 \\
-3 & 1 & 2
\end{pmatrix}
\]

(12 points)

5. For each of the following sets, state whether or not it is a subspace of the given vector space. Give short reasons for your answers.

(a) \(\{(x, y, z) \in \mathbb{R}^3; x - 2y + z = 0\}\) \(\mathbb{R}^3\) is the vector space.

(b) \(\{(x, y, z) \in \mathbb{R}^3; x^2 - 2y^2 + z^2 = 0\}\) \(\mathbb{R}^3\) is the vector space.

(c) \(\left\{\alpha \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} -1 \\ 3 \end{pmatrix}; \alpha, \beta \text{ are any real numbers}\right\}\)

The vector space is \(\mathbb{R}^3\).

(d) The set of polynomials of degree 2 with nonnegative coefficients \(- \text{ i.e., } \{a_2x^2 + a_1x + a_0; a_2, a_1, a_0 \text{ are all nonnegative}\}\). The vector space is the space of all continuous functions defined on the real number line.

(e) The set of all differentiable functions \(f\), with \(f'(0) = 0\). The vector space is the space of all continuous functions defined on the real number line.

(5 points each part)

6. Give a careful proof of the following: suppose \(A\) is an \(n \times n\) matrix; show that \(A\) is a nonsingular matrix if and only if the nullspace of \(A\) is \(\{0\}\). (12 points)

7. If \(A\) is a nonsingular matrix, show that \(\det(A^{-1}) = 1/\det A\). \textbf{Hint: use the equation } \(AA^{-1} = I\). (12 points)