Week-In-Review #2

For each of the following quadratic functions,

(a) find its vertex (is it a max or min?).

(b) find the intervals where the function is increasing/decreasing.

(c) find all real roots of the function.

(d) find the $y$-intercept.

(e) draw a sketch of the function.

1. $f(x) = x^2 - 25$

2. $g(x) = 4x^2 - 8x + 3$
3. $h(x) = x^2 + 2x + 5$

4. $k(x) = -2x^2 + 3x + 10$
5. It is known that 1,500 widgets can be sold when each widget sells for $10 and 3,000 can be sold when each sells for $5. Additionally, it costs the company producing the widgets a total of $300 to produce 10,000 widgets and $50 without producing any widgets. Let $x = \text{the number of widgets made and sold (in hundreds)}. \text{Find the company’s}

(a) price-demand function.

(b) cost function.

(c) revenue function.

(d) profit function.

(e) selling price for widgets in order to maximize their profits. What is the maximum profit?
6. Which of the following are polynomials? If the function is a polynomial, state its degree, leading coefficient, maximum number of turning points, maximum number of possible $x$-intercepts, and the smallest interval where all of the $x$-intercepts can be found.

(a) $f(x) = 7x^3 - ex^2 + \pi$

(b) $g(x) = \sqrt{8x^5}$

(c) $h(x) = 4 - x^2 - \frac{3}{x-4}$

(d) $m(x) = 1 + x^3 - 2x^5 + x^2$

(e) $n(x) = \sqrt{7}x^6 - 9$
7. For the graph below,

(a) How many turning points are on the graph?

(b) What is the minimum degree of a polynomial function that could have the graph?

(c) Is the leading coefficient of the polynomial positive or negative?
8. The following table gives the population (in hundreds of people) of Podunkville, USA for some selected years.

<table>
<thead>
<tr>
<th>Year(x)</th>
<th>1800</th>
<th>1830</th>
<th>1850</th>
<th>1875</th>
<th>1900</th>
<th>1925</th>
<th>1950</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population(y)</td>
<td>25</td>
<td>60</td>
<td>125</td>
<td>350</td>
<td>680</td>
<td>1400</td>
<td>2500</td>
</tr>
</tbody>
</table>

(a) Let $x = 0$ represent the year 1800. Find the best-fitting quadratic, cubic, and quartic models to this data. Round each coefficient to four decimal places, if necessary, in each model. Which of these do you think most accurately models the given data?

(b) Use your unrounded model to predict the population in 1999.

(c) Use your unrounded model to estimate when Podunkville had 9600 people.
9. Find the domain and identify all asymptotes and “holes” in the graphs of the following functions:

(a) \( f(x) = \frac{x^2 - 1}{x^2 - 4} \)

(b) \( g(x) = \frac{-x + 5}{2x^2 + x - 3} \)

(c) \( h(x) = \frac{(x + 4)(x - 2)(4x + 7)}{(x - 2)(x + 3)} \)
10. Find all asymptotes, intercepts and any “holes” which occur in the graph of the following function and then make a sketch of the function.

\[ f(x) = \frac{(-2x + 5)(x - 1)}{(x + 2)(3x - 4)} \]