Week-In-Review #7

1. Find the first three derivatives of the following functions:
   (a) \( f(x) = (5x + 2)^2 \)
   (b) \( g(x) = \sqrt[5]{x^3} \)
   (c) \( h(x) = 4x(2x + 6)^3 \)
For each graph given in questions #2-#5, determine the intervals where

(a) \( f'(x) > 0 \)
(b) \( f'(x) < 0 \)
(c) \( f''(x) > 0 \)
(d) \( f''(x) < 0 \)

and list any \( x \)-values where \( f'(x) = 0 \) or where \( f'(x) \) is undefined.
6. Given \( f'(x) = n(x+2)(x-3)^4(x+5)^3 \), where \( n \) is a function that is always negative, determine critical values of \( f(x) \), intervals where \( f(x) \) is increasing or decreasing and any values of \( x \) where local extrema of \( f(x) \) will occur.

7. Determine the concavity and any inflection points of \( f(x) = 4x - x^4 \), using the second derivative.
For each function in questions #8-#10, find the following information and use it to sketch each function.

(a) Domain of $f(x)$

(b) Intercepts

(c) Asymptotes/Holes

(d) All critical values

(e) All intervals where $f(x)$ is increasing/decreasing

(f) Any local extrema

(g) All intervals where $f(x)$ is concave up/concave down

(h) Any inflection points

8. $f(x) = x^4 - 6x^2 + 5$
9. \( f(x) = \frac{1}{x^2 - 16} \)
10. \( f(x) = \sqrt[3]{2x + 5} \)
11. Sketch the graph of a function that satisfies the following conditions:

- Vertical Asymptotes at \( x = -3, 0, 2 \)
- Horizontal Asymptote at \( y = 0 \)
- \( f'(-2) = 0 \) \( f(-2) = 1 \)
- Inflection point: (1,0)
- \( f'(x) < 0 \) on \( (-\infty, -3), (-3, -2) \) and \( (2, \infty) \)
- \( f'(x) > 0 \) on \( (-2, 0) \) and \( (0, 2) \)
- \( f''(x) < 0 \) on \( (-\infty, -3) \) and \( (0, 1) \)
- \( f''(x) > 0 \) on \( (-3, 0), (1, 2) \) and \( (2, \infty) \)

12. Sketch the graph of a function that satisfies the following conditions:

- Continuous for all reals
- Domain: All reals
- Range: All reals greater than or equal to 4
- \( f'(x) > 0 \) on \( (-5, 0) \) and \( (5, \infty) \)
- \( f'(x) < 0 \) on \( (-\infty, -5) \) and \( (0, 5) \)
- \( f'(0) \) undefined
- \( f''(x) > 0 \) on \( (-\infty, 0) \) and \( (0, \infty) \)
13. Using the given graph of $f''(x)$, sketch possible graphs of $f'(x)$ and $f(x)$. 