Exam II: Night Before Drill

Note: This collection of questions is intended to give you an idea of different types of questions that might be asked on the exam. This is not intended to represent an actual exam. These questions cover Chapter 2, 3.2, 4.1-4.3, and 4.5 in Brief Calculus, The Study of Rates of Change by Armstrong and Davis.

Find the following limits.

1. \[ \lim_{x \to 1} \frac{\sqrt{x+3}}{3^x} \]

2. \[ \lim_{x \to 2} \frac{2x^2 - 4}{3x^2 - 1} \]

3. \[ \lim_{x \to 0} \frac{4x^3 - x^2}{2x^2 - x} \]

4. \[ \lim_{x \to 5} \frac{\sqrt{x} - 5}{x - 25} \]

5. \[ \lim_{x \to \infty} \frac{3x^4 - x + x^3}{4x^3 - x^2 + 5x^4} \]

6. \[ \lim_{x \to \infty} \frac{2x^2 + 1}{x - 6} \]

7. \[ \lim_{x \to -\infty} \frac{x + x^2}{2x^4 - x^2} \]

8. \[ \lim_{x \to -\infty} \frac{e^x - 1}{e^{-x} + 2e^x} \]

9. \[ \lim_{x \to 1} f(x) \text{ if } f(x) = \begin{cases} 2x^2 - 8 & , x < 1 \\ -4\sqrt{x} - 2 & , x > 1 \end{cases} \]
Use the graph below to answer #10 - #18.

10. Find \( \lim_{x \to -1} g(x) \)

11. Find \( \lim_{x \to 2} g(x) \)

12. Find \( \lim_{x \to 1} g(x) \)

13. Find \( \lim_{x \to 2^+} g(x) \)

14. Find \( \lim_{x \to 2^-} g(x) \)

15. \( \lim_{x \to -\infty} g(x) \)

16. \( \lim_{x \to \infty} g(x) \)

17. Find all values of \( x \) where \( g(x) \) is discontinuous and state the continuity condition that is violated.

18. Find all values of \( x \) where \( g(x) \) is not differentiable and give a reason why it is not differentiable.
19. Where is \( f(x) = \begin{cases} 
  x - 6 & , \ x \leq 4 \\
  \frac{x+1}{x-3} & , \ x > 4 
\end{cases} \) discontinuous?

20. Find the value(s) of \( a \) and \( b \), if any exist, that will make \( g(x) = \begin{cases} 
  5a - x & , \ x < 2 \\
  6 & , \ x = 2 \\
  bx^2 + 4 & , \ x > 2 
\end{cases} \) continuous on \((-\infty, \infty)\).

21. Given \( f(x) = x^{1/3} \), where is \( f(x) \)

(a) continuous?

(b) not differentiable?
22. Use the limit definition of derivative to find the derivative of the following functions:

(a) \( f(x) = 2x^2 - x \)
(b) \( g(x) = \frac{x}{x + 1} \)
(c) \( k(x) = \sqrt{x - 3} \)
Find the derivatives of the following functions:

23. \( f(x) = (2x + 1)\sqrt{x^2 + 1} \)

24. \( f(x) = e^{x+e^x} \)

25. \( f(x) = \frac{1}{e^x + e^{-x}} \)

26. \( f(x) = \frac{x + 1}{(x - 2)^3} \)

27. \( f(x) = e^{2x} \ln|2x^3 + x| \)

28. \( f(x) = 4^{x^4+5} \)

29. \( f(x) = \frac{x^2}{3^{2x^4+x}} \)
30. \( f(x) = (x^2 + 6x + 1)^4 \)

31. \( f(x) = xe^x + \ln \left( \frac{3x^2 + 2}{5x - 1} \right) + e^x \)

32. \( f(x) = \log_2(e^{3x} + x^2) \)

33. \( f(x) = \log_3(2x + 1)\sqrt{x^2 + e^{3x^2}} \)

34. Find the instantaneous rate of change of \( f(x) = \sqrt{(x + 7)^3} \) at \( x = 0 \).

35. Find the equation of the tangent line to \( f(x) = \ln(\ln x^2) \) at \( x = e \).
36. If \( C(x) = 5x^2 - 2x + 1000 \) is the cost of producing \( x \) items, in dollars, and \( p = -3x + 30 \) is the price-demand function for the items, find the following:

(a) \( R(x) \)

(b) \( P(x) \)

(c) Marginal profit

(d) The approximate profit from selling the 11\(^{th} \) item

(e) Average profit

(f) Marginal average profit

37. If \( R(x) \) represents a company’s revenue function, what does \( R'(5) \) represent?

38. Given the price-demand function, \( p = \sqrt{1200 - x^2} \), find the demand, as a function of price.
39. A 12-pack of soda regularly sells for $3.99/12-pack. At this price, a local store can sell 150 12-packs. However, when the soda goes on sale for $3.00/12-pack, 200 12-packs are sold. Determine the arc elasticity and interpret.

40. The quantity demanded per week $x$ (in units of a hundred) of the Mikado miniature camera is related to the unit price $p$ (in dollars) by the demand equation $x = \sqrt{400 - 5p}$ $0 \leq p \leq 80$.

   (a) Is the demand elastic or inelastic when $p = 40$? When $p = 60$?
   (b) When is the demand unitary?
   (c) If the unit price is lowered slightly from $60, will the revenue increase or decrease?
   (d) If the unit price is increased slightly from $40, will the revenue increase or decrease?