7.2 - Definition of Probability

The probability of an event, \( P(E) \), is a measure of the proportion of time that the event \( E \) will occur in the long run.

** The probability of an event is ALWAYS a number between 0 and 1. 
\( 0 \leq P(E) \leq 1 \) **

Consider sample space \( S = \{ s_1, s_2, s_3, s_4, \ldots, s_n \} \).

**Simple Events:** those events which consist of exactly one sample point.

* These events are \textit{mutually exclusive}.

\[ P(s_1) + P(s_2) + \cdots + P(s_n) = 1 \]

\[ P(s_i \cup s_j) = P(s_i) + P(s_j) \quad i \neq j \text{ (since mutually exclusive)} \]

Sample spaces in which outcomes are \textit{equally likely} are called \textbf{uniform sample spaces}. The probability for each simple event, \( s_1, s_2, \ldots, s_n \), in such a space is \( \frac{1}{n} \).

**Ex:** A \textit{fair} die is cast and the outcome is recorded.

(a) What is the sample space associated with this experiment?

(b) List all of the simple events.

(c) What is the probability of each simple event?
Probability Distribution: a TABLE which gives the probability associated with each simple event of an experiment

Ex: Find the probability distribution when tossing a fair die once.

Ex: A group was asked to name their favorite color. All of their responses are summarized in the frequency distribution below.

<table>
<thead>
<tr>
<th>Color</th>
<th>Blue</th>
<th>Yellow</th>
<th>Purple</th>
<th>Red</th>
<th>Green</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>45</td>
<td>5</td>
<td>12</td>
<td>10</td>
<td>8</td>
</tr>
</tbody>
</table>

(a) Find the empirical probability distribution for this experiment.

(b) Is this a uniform sample space?

(c) What is the probability that a random person picked from those surveyed said red or blue was his/her favorite color?

(d) What is the probability that a random person picked from those surveyed said burnt orange was his/her favorite color?
**Ex:** A pair of fair dice is cast.

(a) What is the probability that the two dice show different numbers?

(b) What is the probability that at least one 4 is showing?

(c) What is the probability that the sum of the numbers on the two dice is 2? 8? 13?

**Ex:** An experiment consists of picking one letter out of the word OCTOBER.

(a) Write a uniform sample space associated with this experiment.

(b) Write a non-uniform sample space associated with this experiment.