6.4 - Integration by \( u \)-Substitution

Recall: Power Rule for Integration: 
\[
\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)
\]

If we have 
\[
\int [f(x)]^n f'(x) \, dx
\]
we can set \( u = f(x) \) and our integral can be rewritten as 
\[
\int u^n \, du
\]
and therefore, using the power rule, will equal 
\[
\frac{u^{n+1}}{n+1} + C
\]

However, since we started without \( u \)'s, we write the answer with our original variable by substituting \( u = f(x) \) and have 
\[
\frac{[f(x)]^{n+1}}{n+1} + C
\]

So, 
\[
\int [f(x)]^n f'(x) \, dx = \frac{[f(x)]^{n+1}}{n+1} + C \quad (n \neq -1)
\]

REMEMBER...you can check your answer by taking the derivative of your answer.

Ex: Evaluate the following:

(a) \( \int 5(5x + 6)^8 \, dx \)
(b) \[ \int 3x^2 \sqrt[4]{x^3 + 5} \, dx \]

(c) \[ \int \frac{8(x^3 + 1)}{(2x^4 + 8x)^5} \, dx \]

(d) \[ \int \frac{x}{\sqrt[3]{2x^2 + 5}} \, dx \]
(e) $\int (8x + 12)(2x^2 + 6x + 5)^6 \, dx$

(f) $\int x(x + 4)^3 \, dx$

(g) $\int x(x^5 + 1)^2 \, dx$
\[(h) \int_{0}^{2} (2t + 3)^3 \, dt\]

METHOD 1

METHOD 2