Chapter 8 → Calculus of Several Variables

8.1 - Functions of Several Independent Variables

An equation of the form \( z = f(x, y) \) represents a function of two independent variables \( (x \text{ and } y) \) if, for each ordered pair of real numbers \( (x, y) \), the equation determines a unique real number \( z \).

Determining the Domain and Evaluating Functions of Two Variables

Ex: Given \( h(x, y) = x^2 + 2xy + 3\sqrt{y} \), find

(a) \( h(2, 1) \)

(b) \( h(-2, 4) \)

(c) \( h(0, 9) \)

Ex: Find the domains of the following functions.

(a) \( f(x, y) = 5x - 6y \)

(b) \( g(x, y) = \frac{4x}{x - y} \)

(c) \( h(x, y) = e^{2x} - x \ln (y + 1) \)
Ex: Suppose a company sells two types of candy bars: plain chocolate and chocolate with almonds. Let $x$ = the number of chocolate with almond bars sold/day and $y$ = the number of plain bars sold/day. The price per bar is given by $p = 9 - 2x + y$ for the plain bars and $q = 12 + x - 2y$ for the almond bars.

(a) Find the revenue function for the company.

(b) Suppose the cost function for the company is given to be $C(x, y) = 24 + 3x + 6y$. What is the profit equation?

(c) Find the profit/loss of selling 6 almond bars and 5 plain bars.

Cartesian Coordinate System (in 3D)

To graph a function $z = f(x, y)$, we must use a 3D space.
Graphing Planes