1.3 - Linear Functions and Straight Lines

Intercepts

\textbf{x-intercept}: where a graph crosses the \( x \) axis \( (\implies y = 0) \)
\textbf{y-intercept}: where a graph crosses the \( y \) axis \( (\implies x = 0) \)

Linear Functions \& Equations

\textbf{Def}: A function \( f \) is a \textbf{linear function} if

\[ f(x) = mx + b \quad m \neq 0 \]

where \( m \) and \( b \) are real numbers.

\textbf{Def}: If \( m = 0 \), then \( f \) is called a \textbf{constant function}, \( f(x) = b \).

\textbf{Def}: The \textbf{slope} of a line, denoted by \( m \), measures the “steepness” of the line relative to the \( x \)-axis. Given two points on a line \( (x_1, y_1) \) and \( (x_2, y_2) \), the slope of the line is computed by

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \quad (x_1 \neq x_2) \]

\textbf{Ex}: Find the slope of the line passing through the points \((1,-2)\) and \((-4, 8)\).
Line Properties

<table>
<thead>
<tr>
<th>Type</th>
<th>Slope</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal</td>
<td>$m = 0$</td>
<td></td>
</tr>
<tr>
<td>Vertical</td>
<td>$m$ undefined</td>
<td></td>
</tr>
<tr>
<td>Rising (Increasing)</td>
<td>$m &gt; 0$</td>
<td></td>
</tr>
<tr>
<td>Falling (Decreasing)</td>
<td>$m &lt; 0$</td>
<td></td>
</tr>
</tbody>
</table>

Equations of Lines

- **Standard Form:** $Ax + By = C$  
  $A$ and $B$ not both 0.
- **Slope-Intercept Form:** $f(x) = y = mx + b$  
  Line with slope, $m$, & y-int at $(0, b)$.
- **Point-Slope Form:** $y - y_1 = m(x - x_1)$  
  Line with slope, $m$ & thru pt $(x_1, y_1)$.
- **Vertical Line:** $x = a$  
  $a$ is a constant value; $m$ undefined
- **Horizontal Line:** $y = b$  
  $b$ is a constant value; $m = 0$

Ex: Find the $x$- and $y$-intercepts of $y = \frac{4}{7}(x - 7) + 1$. 

Ex: Write an equation for each line described.

(a) The line passing through (1,2) and (-4,3).

(b) The vertical line passing through (9,-7).

(c) The line having an $x$-intercept at 4 and a $y$-intercept at 5.

(d) A linear function, $f(x)$, in which $f(0) = 8$ and slope is $1/2$. 
Applications of Linear Functions

**Linear Depreciation Function:** The value of an object at any given time, assuming it is losing value (*depreciating*) at a constant rate over time.

\[ V(t) = mt + b \]

**Ex:** Kathryn bought a new car in 1999 for $20,500. In 2002, it is only worth $13,285. Assuming it depreciates at a constant rate, what will Kathryn’s car be worth in 2006?

**Linear Cost Function:** The total cost of producing \( x \) items - taking into account both variable and fixed costs.

\[ C(x) = cx + F \]

**Ex:** KB & Co. is manufacturing insulated mugs. The company has monthly fixed costs of $1500 and there is a total monthly cost of $1800 when producing 100 mugs. Find the linear cost function for KB & Co.
Linear Supply and Demand:

**Ex:** At a price of $100 per calculator, the annual supply and demand for calculators in a particular market are 1200 and 900, respectively. When the price rises to $110, the supply increases to 1500 calculators while the demand decreases to 800 calculators.

(a) Assuming that the price-supply and the price-demand equations are linear, find the equations for each.

(b) Find the equilibrium point for the calculator market described.
**Linear Regression:** Finding the best linear function fitting a given set of data.

1. Standardize the data.

2. Enter the data into $L_1$ and $L_2$ by pressing $\text{STAT}$ and selecting $1:\text{Edit}$.

3. Graph the data points in a scatterplot if you wish.

4. Find the equation of the best linear model.
   Press $\text{STAT}$, move right and select $\text{CALC}$ and then go down to $4:\text{LinReg}(ax+b)$ for a linear model ($y = ax + b$)

5. Store your model in the calculator, so you can compare models to one another and make predictions with your model. To do this, after making your model selection described above, press $\text{VARS}$ move right and select $\text{Y-VARS}$, choose option $1:\text{Function}$ and then choose a function name.

**Ex:** The following table represents price-demand data where $p =$ the wholesale price per widget at which $x$ thousand widgets are sold.

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>88</td>
<td>67</td>
<td>54</td>
<td>36</td>
</tr>
</tbody>
</table>

(a) Find the equation of the best fitting line to the data (Round each coefficient to four decimal places, if necessary.)

(b) Use your unrounded model to predict the price (to the nearest cent) at which 9,600 widgets would be demanded.

(c) Use your unrounded model to predict the number of widgets (to the nearest widget) that would be demanded at a price of $12$. 