1.4 - Quadratic Functions

Properties of Quadratic Functions

Def: A function of the form

\[ f(x) = ax^2 + bx + c \]

is called a quadratic function, where \( a, b, \) and \( c \) are real numbers and \( a \neq 0 \).

\[ a > 0 \quad \text{and} \quad a < 0 \]

Def: The maximum or minimum of a quadratic function \( f(x) \), whose graph is known as a parabola, is called its vertex and has coordinates

\[ (x, y) = (h, k) = \left( \frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right) \]

Ex: Given \( f(x) = -2x^2 + 4x + 3 \), determine

(a) the direction the parabola opens.

(b) the \( y \)-intercept.

(c) the vertex (is it a max or min?).

(d) the domain of \( f(x) \).

(e) the range of \( f(x) \).

(f) the interval(s) where \( f(x) \) is increasing/decreasing.
Determining Zeros of a Quadratic Function

Def: The real zeros (or roots) of a function are its $x$-intercepts.

To find the EXACT zeros of a quadratic function:

1. Factor, set each factor equal to zero, and solve for $x$.
2. Use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

To approximate the zeros of any function, you can use your calculator:

1. Graph your function on a window where the zeros are visible.
2. Press $\text{2nd TRACE}$ and select option 2:$\text{zero}$.
3. Follow the directions on the screen, using your left and right arrow keys to move along the curve.

Ex: Find the EXACT zeros of the following functions:

(a) $f(x) = 6x^2 - 7x - 3$

(b) $g(x) = -2x^2 + 4x + 3$

Ex: Solve $x^2 + 3x = 40$
Applications

Ex: A resort has 20 rooms and can rent all of them at a price of $250/day. However, management knows that one room becomes vacant with every $20 increase in room price. Also, the cost to maintain rooms is $50/day per room, plus an additional $1000/day.

(a) Find the revenue, cost, and profit functions for the resort.
(b) How many rooms must be rented in order for the resort to break-even?
(c) How many rooms must be rented in order to maximize profit?
(d) What rent should be charged in order to maximize profit?
Quadratic Regression: Finding the best quadratic function fitting a given set of data.

1. Standardize the data.

2. Enter the data into $L_1$ and $L_2$ by pressing $\text{STAT}$ and selecting $1:Edit$.

3. Graph the data points in a scatterplot if you wish.

4. Find the equation of the best quadratic model.
   Press $\text{STAT}$, move right and select $\text{CALC}$ and then go down to $5:QuadReg$ for a quadratic model ($y = ax^2 + bx + c$).

5. Store your model in the calculator, so you can compare models to one another and make predictions with your model. To do this, after making your model selection described above, press $\text{VARS}$ move right and select $\text{Y-VARS}$, choose option $1:Function$ and then choose a function name.

**Ex: (#53 data from B.Z.B.)** An automobile tire manufacturer collected the following data relating tire pressure, $x$, (in pounds per square inch) and mileage, $y$ (in 1000s of miles).

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<th>30</th>
<th>32</th>
<th>34</th>
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<tbody>
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<tr>
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<td>52</td>
<td>55</td>
<td>51</td>
<td>47</td>
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(a) Find the equation of the best fitting quadratic function to the data (Round each coefficient to four decimal places, if necessary.)

(b) What mileage (to the nearest mile) would you estimate the tire to have if the tire pressure was set at 31 pounds per square inch?