3.4 - Power Rule and Basic Differentiation Properties

**Derivative Notation for** \( y = f(x) \): \( f'(x) = y' = \frac{dy}{dx} = \frac{d}{dx}[f(x)] \)

**Constant Function Rule:** For any constant \( k \), if \( f(x) = k \), then \( f'(x) = 0 \).

**Ex:** Differentiate the following:

(a) \( f(x) = 2 \)

(b) \( g(x) = e \)

(c) \( h(x) = \sqrt[3]{5} \)

**Power Rule:** If \( f(x) = x^n \), where \( n \) is any real number, \( f'(x) = nx^{n-1} \).

**Ex:** Differentiate the following:

(a) \( f(x) = x^3 \)

(b) \( g(t) = \sqrt[3]{t^3} \)

(c) \( y = \frac{1}{x^4} \)

(d) \( y = x^{2.47} \)

**Constant Multiple Rule:** If \( f(x) = kg(x) \), where \( k \) is any real number, \( f'(x) = kg'(x) \).

**Ex:** Differentiate the following:

(a) \( f(x) = 5x^{-7} \)

(b) \( g(x) = \frac{3}{4} \sqrt{x} \)

(c) \( y = \frac{4}{x} \)
Sum and Difference Rule: If \( h(x) = f(x) \pm g(x) \), where \( f \) and \( g \) are both differentiable functions, then \( h'(x) = f'(x) \pm g'(x) \).

Ex: Differentiate the following:

(a) \( y = 4x^3 + 5x^2 - 7 \)

(b) \( f(t) = \sqrt{5}t - t^{1/2} + e \)

(c) \( g(x) = x^{-3} + \frac{2}{5}x^{-1} + 2x^{1/4} \)

(d) \( k(x) = \frac{3x^4 + 2x^2 - 4 + x}{x} \)

(e) \( m(x) = \frac{2x^2 + x^3}{2\sqrt{x}} \)

Ex: If a watermelon is dropped from a building 600 feet tall, its height above the ground (in feet) after \( t \) seconds is given by \( s(t) = 600 - 16t^2 \).

(a) Compute \( s'(t) \) and give its units. What does \( s'(t) \) measure?

(b) Compute \( s(1) \) and \( s'(1) \) and interpret each.

(c) When does the watermelon hit the ground?
Ex: Find the value(s) of \( x \) where \( f(x) = 6x^5 - 5x^3 \) has a horizontal tangent line.