3.7 - Marginal Analysis in Business and Economics

Def: Marginal analysis is the study of the amount of change in the dependent variable that results from a single unit change in the independent variable.

Ex: Suppose the total cost (in $) of producing \( x \) widgets is given by \( C(x) = 0.5x^2 - 12x + 100 \).

(a) Find \( C(16) - C(15) \) and interpret.

(b) Find \( C'(15) \), interpret, and compare to the value in (a).

Def: If \( x \) represents the number of units of a product produced in some time interval, then

- Total Cost = \( C(x) \) and Marginal Cost = \( MC(x) = C'(x) \)
- Total Revenue = \( R(x) \) and Marginal Revenue = \( MR(x) = R'(x) \)
- Total Profit = \( P(x) = R(x) - C(x) \) and Marginal Profit = \( MP(x) = P'(x) \)

Marginal cost/revenue/profit functions are the IROC of cost/revenue/profit, relative to production at a given production level. Marginal cost/revenue/profit approximate the exact cost/revenue/profit obtained from producing one more item \( \Rightarrow \) the approximate cost/revenue/profit of the next item.
Ex: A toy manufacturer has just produced a new doll that sells to wholesalers for $25. The cost, in dollars, to produce $x$ dolls is given by $C(x) = -0.05x^2 + 2x + 1500$.

(a) Find the profit function, $P(x)$.

(b) Find $P(100)$ and interpret.

(c) Find $MP(100) = P'(100)$ and interpret.
**Def:** If $x$ represents the number of units of a product produced in some time interval, then

- **Average Cost** $= AC(x) = \frac{C(x)}{x}$ and **Marginal Average Cost** $= MAC(x) = AC'(x)$

- **Average Revenue** $= AR(x) = \frac{R(x)}{x}$ and **Marginal Average Revenue** $= MAR(x) = AR'(x)$

- **Average Profit** $= AP(x) = \frac{P(x)}{x}$ and **Marginal Average Profit** $= MAP(x) = AP'(x)$

**Ex: (#10)** The total profit (in dollars) from the sale of $x$ charcoal grills is

$$P(x) = 20x - 0.02x^2 - 320 \quad 0 \leq x \leq 1000$$

(a) Find the average profit per grill if 40 grills are produced.

(b) Find the marginal average profit at a production level of 40 grills and interpret.

(c) Use the results from (a) and (b) to estimate the average profit per grill if 41 grills are produced.