4.1 - First Derivatives and Graphs

Increasing and Decreasing Functions: On an open interval $(a, b)$ on which $f(x)$ is differentiable and continuous

(a) $f(x)$ is increasing on $(a, b)$ if $f'(x) > 0$ on $(a, b)$.

(b) $f(x)$ is decreasing on $(a, b)$ if $f'(x) < 0$ on $(a, b)$.

(c) $f(x)$ is constant on $(a, b)$ if $f'(x) = 0$ on $(a, b)$.

Ex: Using the graph of $f(x)$ below, answer the following questions.

(a) On what intervals is $f'(x) > 0$ and what does this say about $f(x)$?

(b) On what intervals is $f'(x) < 0$ and what does this say about $f(x)$?

(c) On what intervals is $f'(x) = 0$ and what does this say about $f(x)$?
**Def:** A critical value for \( f(x) \) is an \( x \)-value in the domain of \( f(x) \) for which

1. \( f'(x) = 0 \) or
2. \( f'(x) \) is undefined (DNE)

(NOTE: Critical values will be partition numbers of our sign chart for the first derivative. Also, any \( x \)-value not in the domain of the function will also be a partition number of our sign chart for the first derivative.)

**Ex:** Find the critical values and partition numbers for the following functions and then construct a sign chart for the first derivative to determine where the function is increasing/decreasing.

(a) \( f(x) = x^3 + 3x^2 - 9x + 3 \)

(b) \( g(x) = \sqrt[3]{x} \)

(c) \( h(x) = \frac{1}{x} \)

**First Derivative Test**

Suppose \( x = c \) is a critical value of \( f(x) \).

1. If \( f'(x) \) changes from (+) to (-) at \( x = c \), then we have that \( f(x) \) is \( \nearrow \searrow \) and at \( x = c \) there is a local maximum.
2. If \( f'(x) \) changes from (-) to (+) at \( x = c \), then we have that \( f(x) \) is \( \searrow \nearrow \) and at \( x = c \) there is a local minimum.
3. If the sign of \( f'(x) \) is the same on both sides of \( x = c \), then at \( x = c \) there is neither a local maximum nor a local minimum.

(NOTE: Local extrema means all local maxima and local minima. All local extrema will occur at critical values, but not all critical values will produce local extrema.)
Ex: Determine the intervals where the following functions are increasing and decreasing and locate any points where local extrema occur.

(a) \( f(x) = x^3 + 3x^2 - 9x + 3 \)

(b) \( f(x) = \frac{x^2}{x - 1} \)

Ex: Given \( f'(x) = p(x + 2)^2(x - 5)^3(x - 10) \) where \( p \) is a function that is always positive, find all critical values of \( f(x) \), all intervals where \( f(x) \) is increasing and decreasing, and all places where local extrema occur.
Ex: (#50) If it is known that \( f(x) \) is a continuous function over \( (-\infty, \infty) \) and you are given the information below, sketch a graph of \( f(x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>-3</td>
<td>0</td>
<td>2</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

Ex: Use the given graph to answer the following questions:

(a) If the graph is that of \( f(x) \), where is \( f'(x) > 0? \) \( f'(x) < 0? \) \( f'(x) = 0? \)

(b) If the graph is that of \( f'(x) \), where is \( f(x) \) increasing? Decreasing?

(c) If the graph is that of \( f'(x) \), where does \( f(x) \) have local extrema?