Exam III Review Problems  
Fall 2011  

Note: Not every topic is covered in this review.  
Please also take a look at the previous Week-in-Reviews for more practice problems. 

1. Determine whether the following statements are True or False. 

(a) The number of minutes it takes you to use an ATM machine is an infinite discrete random variable. 

\[ X > 0 \Rightarrow \text{continuous} \quad \text{FALSE} \]

(b) An experiment consists of drawing cards, without replacement, from a standard 52-card deck until all 4 Aces have been drawn. Let \( X \) represent the number of draws needed. \( X \) can be any value in the set \( \{1, 2, 3, \ldots, 52\} \). 

\[ X = 4, 5, 6, \ldots, 52 \quad \text{FALSE} \] 
\[ \text{finite discrete} \]

(c) The number of cadets in a class of 100 students is a finite discrete random variable. 

\[ X = 0, 1, 2, \ldots, 100 \Rightarrow \text{finite discrete} \quad \text{TRUE} \]
The odds of drawing an Ace from a standard 52-card deck on the second draw, if cards are drawn without replacement and it is known that the first card drawn was a king, are 4 to 47.

\[
P(A_2 | K_1) = \frac{4}{51}
\]

\[
P(A_2^c | K_1) = \frac{47}{51}
\]

\[
\text{Odds} = \frac{\frac{4}{51}}{\frac{47}{51}} = \frac{4}{47} \Rightarrow 4 \text{ to } 47
\]

True

(e) If the odds against an event \( E \) occurring are 3 to 19, then \( P(E) = \frac{19}{22} \)

\[
P(E) = \frac{19}{3+19} = \frac{19}{22}
\]

\[
P(E^c) = \frac{3}{22}
\]

True
(f) The total area of a probability histogram is equal to 1.

\[
\text{width} = 1
\]

\[\text{height} = \text{prob. of r.v. value}\]

\[
\text{Area} = 1 (\text{prob}) = \text{prob}
\]

All areas should sum to 1

(g) An experiment with \( n \) outcomes will have \( 2^n \) simple events.

\[
S = \{ s_1, s_2, \ldots, s_n \}
\]

\( n \) outcomes

Events = subsets of \( S \)

Simple events = subsets containing exactly one sample point

\[
\Rightarrow \{ \{s_1\}, \{s_2\}, \ldots, \{s_n\} \}
\]

\( n \) simple events

FALSE
2. There are 130 boxes of Cheerios in a grocery store. The following table tells you how many boxes had a certain number of Cheerios in them.

<table>
<thead>
<tr>
<th>L1</th>
<th>X= # of Cheerios</th>
<th>510</th>
<th>480</th>
<th>467</th>
<th>434</th>
<th>521</th>
<th>555</th>
</tr>
</thead>
<tbody>
<tr>
<td>L2</td>
<td>Freq # of Boxes</td>
<td>23</td>
<td>17</td>
<td>40</td>
<td>30</td>
<td>9</td>
<td>11</td>
</tr>
</tbody>
</table>

(a) Place an X in the row of the table which represents the variable being measured.
(b) Find the mean, median, mode, standard deviation, and variance for X.

\[ \text{Mean} = \bar{x} = \frac{31192}{65} \approx 479.8769 \text{ Cheerios} \]

\[ \text{Median} = \text{Med} = 467 \text{ Cheerios} \]

\[ \text{Std Dev} = s_x \approx 35.9425 \text{ Cheerios} \]

\[ \text{Var}(x) = (s_x)^2 = \frac{5459116}{4225} \approx 1291.8618 \text{ (Cheerios}^2) \]

(c) Find the probability distribution of \( X \).

<table>
<thead>
<tr>
<th>X</th>
<th>prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>510</td>
<td>( \frac{23}{130} )</td>
</tr>
<tr>
<td>480</td>
<td>( \frac{17}{130} )</td>
</tr>
<tr>
<td>467</td>
<td>( \frac{40}{130} )</td>
</tr>
<tr>
<td>434</td>
<td>( \frac{30}{130} )</td>
</tr>
<tr>
<td>521</td>
<td>( \frac{9}{130} )</td>
</tr>
<tr>
<td>555</td>
<td>( \frac{11}{130} )</td>
</tr>
</tbody>
</table>

\[ \text{Sum} = 1 \]

(d) Find \( P(X \geq 500) = \frac{23}{130} + \frac{9}{130} + \frac{11}{130} = \frac{43}{130} \]
3. Suppose you pay $10 to roll two fair six-sided dice and sum the numbers that show. You win twice what you paid if a 7 or 11 shows up. You lose what you paid if a 2, 3, or 12 shows up. For anything else that shows up, you win $5. Let $X$ be your net winnings.

(a) What are your expected net winnings?

$$E(X) = ?$$

$$X = \text{net win} \begin{array}{c|c|c} 7, 11 & 2, 3, 12 & 5 - 10 \\ \hline \text{prob} & \frac{2}{36} & \frac{4}{36} & \frac{24}{36} \\ & 1 - \left( \frac{2}{36} + \frac{4}{36} \right) \\ & \frac{30}{36} \\ & \frac{5}{6} \\
\end{array}$$

$$E(X) = 10 \left( \frac{8}{36} \right) + (-10) \left( \frac{4}{36} \right) + (-5) \left( \frac{24}{36} \right)$$

$$= \frac{-80}{36}$$

$$\approx -\$2.22$$
(b) How much should be charged to make this a fair game? $\Rightarrow E(\text{net win}) = 0$

<table>
<thead>
<tr>
<th>$X_{\text{net win}}$</th>
<th>$2p - p$</th>
<th>$0 - p$</th>
<th>$5 - p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob</td>
<td>$\frac{8}{36}$</td>
<td>$\frac{4}{36}$</td>
<td>$\frac{24}{36}$</td>
</tr>
</tbody>
</table>

$$E(x) = (p)(\frac{8}{36}) + (-p)(\frac{4}{36}) + (5-p)(\frac{24}{36}) = 0 \text{ to make a fair game}$$

$$\frac{8}{36}p - \frac{4}{36}p + \frac{10}{3} - \frac{24}{36}p = 0$$

$$-\frac{20}{36}p + \frac{10}{3} = 0$$

$$p = \frac{10}{3} \Rightarrow \text{Charge $\$6}$$
4. Determine whether or not the following experiments are binomial.
(a) Roll a pair of fair six-sided dice 10 times and observe whether or not a sum of 2 is rolled.

\[
\begin{align*}
10 \text{ trials} \\
\text{Sum}=2 \text{ or } \text{Sum} \neq 2 \\
p(\text{Sum}=2) = \frac{1}{36} \\
\text{Indep} \checkmark
\end{align*}
\]

\[
\text{Yes, binomial.}
\]

(b) Roll a fair six-sided die 8 times and note the number rolled.

\[
\begin{align*}
8 \text{ trials} \\
6 \text{ outcomes} \rightarrow \text{Not binomial}
\end{align*}
\]

(c) Toss a fair coin until a head is tossed.

\[
\text{Not a fixed # of trials} \Rightarrow \text{Not binomial}
\]
(d) Pick 4 marbles, in succession without replacement, from a box with 4 red and 5 green marbles and observe the color of the marble picked.

4 trials
Red or Green

\[ P(\text{Red}_1) = \frac{4}{9} \]
\[ P(\text{Red}_2 | \text{G}_1) = \frac{4}{8} \]
\[ P(\text{Red}_2 | \text{R}_1) = \frac{3}{8} \]  \( \therefore \) trials are not indep \( \Rightarrow \) not binomial

(e) Pick a marble from Box 1 containing 4 red and 5 green marbles and observe the color of the marble picked. Pick a marble from Box 2 containing 3 red and 6 green marbles and observe the color of the marble picked.

2 trials
Red or Green

\[ P(\text{Red}_1) = \frac{4}{9} \]  \( \therefore \) Prob change \( \Rightarrow \) not binomial
\[ P(\text{Red}_2) = \frac{3}{9} \]
5. 15% of a given population is left-handed. A sample of 50 people from the population is selected at random. What is the probability that

(a) Exactly 8 people are left-handed?

\[ P(X=8) = \text{binompdf}(50, 0.15, 8) \approx 0.1493 \]

(b) At most 15 people are left-handed?

\[ P(X \leq 15) = P(0 \leq X \leq 15) = \text{binomcdf}(50, 0.15, 15) \approx 0.9981 \]

(c) More than 11 people are left-handed?

\[ P(X > 11) = P(12 \leq X \leq 50) = \text{binomcdf}(50, 0.15, 50) - \text{binomcdf}(50, 0.15, 11) \approx 0.0628 \]

(d) Between 6 and 20 people are left-handed?

\[ P(6 < X < 20) = P(7 \leq X \leq 19) = \text{binomcdf}(50, 0.15, 19) - \text{binomcdf}(50, 0.15, 6) \approx 0.6387 \]
6. You roll a weighted six-sided die 500 times.
   The die is weighted such that the probability of the die showing a 1 is 0.8.
   (a) What’s the probability that exactly 408 ones are rolled?
   \[ p(X = 408) = \text{binompdf}(500, 0.8, 408) \approx 0.0306 \]
   (b) What’s the probability that at least 375 ones are rolled?
   \[ p(X \geq 375) = 1 - p(X < 375) = 1 - \text{binomcdf}(500, 0.8, 374) \approx 0.9973 \]
   (c) What’s the probability that fewer than 390 ones are rolled?
   \[ p(X < 390) = \text{binomcdf}(500, 0.8, 389) \approx 0.1210 \]
   (d) How many ones should you expect to roll?
   \[ E(X) = np = 500(0.8) = 400 \text{ ones} \]
   (e) What is the variance and standard deviation in the number of ones rolled?
   \[ \text{Var}(X) = npq = 500(0.8)(0.2) = 80 \]
   \[ \text{Std Dev} = \sqrt{npq} = \sqrt{80} \]
7. The table below gives the results of a survey conducted at Idea University regarding whether or not students enjoy studying math and/or history. Use the information to answer the questions which follow.

<table>
<thead>
<tr>
<th></th>
<th>Enjoy Only Math</th>
<th>Enjoy Only History</th>
<th>Enjoy Math and History</th>
<th>Enjoy Neither</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(T)</td>
<td>(H)</td>
<td>(B)</td>
<td>(N)</td>
<td></td>
</tr>
<tr>
<td>Females (F)</td>
<td>35</td>
<td>20</td>
<td>15</td>
<td>10</td>
<td>80</td>
</tr>
<tr>
<td>Males (M)</td>
<td>40</td>
<td>45</td>
<td>25</td>
<td>5</td>
<td>115</td>
</tr>
<tr>
<td>Total</td>
<td>75</td>
<td>65</td>
<td>40</td>
<td>15</td>
<td>195</td>
</tr>
</tbody>
</table>

(a) What is the probability that one of the surveyed students picked at random is a male who enjoys only history?

\[ P(M \land H) = \frac{45}{195} \]

(b) What is the probability that one of the surveyed students picked at random enjoys history or math?

\[ P(T \cup H \cup B) = \frac{75 + 65 + 40}{195} = \frac{180}{195} \]

(c) If a male is selected at random, what is the probability that he enjoys studying history?

\[ P(H \cup B | M) = \frac{45 + 25}{115} = \frac{70}{115} \]

(d) What is the probability that a randomly selected female does not enjoy studying either subject?

\[ P(N | F) = \frac{10}{80} \]

(e) What is the probability that a randomly selected student who enjoys studying math also enjoys studying history?

\[ P(B | T \cup B) = \frac{40}{75 + 40} = \frac{40}{115} \]
8. You have a uniform sample space $S = \{s_1, s_2, \ldots, s_6\}$ for an experiment with events $A = \{s_1, s_2, s_3, s_4\}$ and $B = \{s_2, s_4, s_6\}$.

(a) Draw a probability distribution for this experiment.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$s_4$</th>
<th>$s_5$</th>
<th>$s_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob.</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
<td>$\frac{1}{6}$</td>
</tr>
</tbody>
</table>

(b) Compute $P(A \cap B)$

$$A \cap B = \{s_2, s_4\} \quad \Rightarrow \quad P(A \cap B) = P(s_2) + P(s_4) = \frac{2}{6}$$

(c) Compute $P(A \cup B)$

$$A \cup B = \{s_1, s_2, s_3, s_4, s_6\} \quad \Rightarrow \quad P(A \cup B) = \frac{5}{6}$$

(d) Compute $P(A|B^c)$

$$A \cap B^c = \{s_1, s_3\} \quad B^c = \{s_1, s_2, s_5\}$$

$$P(A|B^c) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{\frac{2}{6}}{\frac{4}{6}} = \frac{\frac{2}{6}}{\frac{2}{3}} = \frac{2}{3}$$

(e) Are $A$ and $B$ independent events? Why or why not? (Use correct mathematical justification.)

$$\text{Ind} \iff P(A \cap B) = P(A)P(B)$$

$$\frac{2}{6} = \left(\frac{4}{6}\right)\left(\frac{2}{3}\right) \quad \Rightarrow \quad \frac{2}{6} = \frac{8}{18} \quad \Rightarrow \quad \frac{1}{3} = \frac{1}{3} \quad \Rightarrow \ A \cap B \text{ independent}$$
9. Let $E$ and $F$ be two events with $P(E) = 0.35$, $P(F) = 0.55$, and $P(E \cap F^c) = 0.15$.

Answer the following questions.

(a) $P(E \cap F) = \boxed{0.2}$

(b) $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.55} = \boxed{0.3636}$

(c) $P(E \cup F) = a + b + c = \boxed{0.7}$

(d) Compute the probability of exactly one of these events ($E$ or $F$) occurring:

\[ a + c = P(E \cap F^c) + P(F \cap E^c) = 0.15 + 0.35 = \boxed{0.5} \]

(e) Are $E$ and $F$ mutually exclusive? Why or why not?

$E \cup F$ cannot occur at the same time

$E \cap F = \emptyset$; $P(E \cap F) = 0$; $P(E \cup F) = P(E) + P(F)$

\[ P(E \cap F) = 0.2 \neq 0 \]

\[ \Rightarrow \text{not mutually excl.} \]

(f) Are $E$ and $F$ independent? Why or why not?

\[ \text{Indep} \iff P(E \cap F) = P(E)P(F) \]

\[ 0.2 \neq (0.35)(0.55) \]

\[ \Rightarrow 0.2 \neq 0.1925 \Rightarrow E \cup F \text{ NOT indep (dependent)} \]
10. A box contains 21 candles: 10 white, 3 red, 6 green, and 2 navy. You lose electricity and randomly select 6 candles from the box to use for light. What’s the probability that you select

(a) Exactly 3 white candles?

\[
\begin{align*}
&= \frac{\binom{10}{3} \binom{11}{3}}{\binom{21}{6}} \\
&= \frac{825}{2261}
\end{align*}
\]

(b) At least 2 green candles?

\[
P(\text{at least 2G}) = 1 - P(\text{less than 2G})
\]

\[
= 1 - \left[ \frac{\binom{6}{0} \binom{15}{6} + \binom{6}{1} \binom{15}{5}}{\binom{21}{6}} \right]
\]

\[
= \frac{4463}{7752}
\]

(c) Exactly 2 red \(\bigcirc\) or exactly 3 green candles?

\[
P(\text{2R} + \text{3G}) = P\left(\binom{2}{4,0} + \binom{3}{3,0}\right) - P\left(\binom{2}{3,2}\right)
\]

\[
= \frac{\binom{3}{2} \binom{18}{4} + \binom{6}{3} \binom{15}{3} - \binom{3}{2} \binom{6}{3} \binom{12}{1}}{\binom{21}{6}}
\]

\[
= \frac{2195}{6783}
\]
11. A math class has a row of 14 students with 8 freshmen, 2 juniors, and 4 seniors. What is the probability that they are seated with all students of the same classification sitting next to one another?

\[
\frac{3! (8! 2! 4!)}{14!}
\]

\[\approx 1.332 \times 10^{-4}\]
12. Use the partially drawn tree to the right to compute the following probabilities.

(a) \( P(C|B) = \frac{0.05}{0.6} \)

(b) \( P(B \cap C) = \frac{0.4 \times 0.05}{0.6} = \frac{0.02}{0.6} = \frac{1}{7} \)

(c) \( P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{0.4 \times 0.05}{0.6 \times 0.2 + 0.4 \times 0.05} = \frac{0.02}{0.14} = \frac{1}{7} \)

(d) \( P(C^c) = 1 - P(C) = 1 - [0.14] = 0.86 \)
13. A chef’s school is 60% male. Seventy percent of the males and 90% of the females like eating beef Wellington for dinner. A student of the school is selected at random.

(a) Draw a tree diagram representing this situation.

(b) What is the probability that the student is \(\text{male}\) or \(\text{likes eating beef Wellington}\) for dinner?

\[
\begin{align*}
P(M \cup W) &= P(M) + P(W) - P(M \cap W) \\
&= 0.6 + [0.6(0.7) + 0.4(0.9)] - 0.6(0.7) \\
&= 0.96
\end{align*}
\]

(c) If the student likes eating beef Wellington for dinner, what is the probability that the student is \(\text{female}\)?

\[
P(F \mid W) = \frac{P(F \cap W)}{P(W)} = \frac{0.4(0.9)}{0.6(0.7) + 0.4(0.9)} = \frac{6}{13}
\]

(d) What percentage of the students like eating beef Wellington for dinner?

\[
P(W) = 0.6(0.7) + 0.4(0.9) = 0.78 \Rightarrow 78\%
\]
14. The weather forecaster on Ch. 5 is correct 90% of the time and the forecaster on Ch. 9 is correct 65% of the time. If the forecasters make their weather predictions independently of each other, what is the probability that on a given occasion

(a) Exactly one of the forecasters is correct?

\[ P(A \cap B^c) + P(A^c \cap B) \]
\[ = 0.9(0.35) + 0.1(0.65) \]
\[ = 0.38 \]

(b) At least one of the forecasters is correct?

\[ P(\text{at least one correct}) = 1 - P(\text{both wrong}) \]
\[ = 1 - 0.1(0.35) \]
\[ = 0.965 \]