33. ∀ τ ∈ S_n. and π is odd. Then π = o ◦ (ω^−1 π)
    where ω o^−1 is odd, π is odd =⇒ o^−1 π is even ∈ A

35. If o is a cycle of length n, then o^r is also a cycle if and only if gcd(r, n) = 1, or n | r.
    Let G be the gp generated by o. Then G = <o> ≅ Z_n.
    If o^r is a cycle,
    Case 1. o^r = e =⇒ n | r.
    Case 2. o^r ≠ e. Then clearly o^r has no fixed pts.
    Then o^r is a cycle =⇒ o^r is a cycle of length n
    =⇒ |<o^r>| = n. i.e. o^r is a generator of G

By Thm 6.14, it is only possible when gcd(r, n) = 1.
(Case 1 is needed since in our book, e is by definition a cycle.)

§10.

19. False: d. f. i.

∀ b ∈ G. H < G.

20. Not possible. For abelian G, bH = \{bh | h ∈ H\} = \{hb | h ∈ H\} = Hb

21. Take subgroup H = G

22. Take subgroup H = Z_3.


28. ∀ g, ∀ h ∈ H. g^−1 h g ∈ H =⇒ Let \( \tilde{h} = g^−1 h g \)
    =⇒ g^\tilde{h} = h g ⇒ h g ∈ gH.
Hence Hg ∈ gH. (Cont'd in the next pg)

But both Hg and gH have one correspondence with H.
Conversely, take \( g_1 = g^{-1} \). \( g_1, h, g \in H \Rightarrow g h g^{-1} \in H \)
\( \Rightarrow g h g^{-1} = h_2 \Rightarrow g h = h_2 g \Rightarrow g h \in H g \Rightarrow g H \subseteq H g \)
Combining above, \( g H = H g \) \( \forall g \).

29. \( \forall g, H g \) is the only left coset of \( H \) that contains \( g \).
\( H g \) is the only right coset of \( H \) that contains \( g \).
If left cosets give the same partition as right cosets.
Then \( g H = H g \Rightarrow \exists h \in H \) s.t.
\( h g = g h \)
\( \Rightarrow g^{-1} h g = h \in H \).

34. \( \forall H \leq G, H \neq G \). Then \( |H| < p^2 \), and \( |H| = 1 \) \( \Rightarrow \)
\( |H| = 1 \text{ or } p \text{ or } q \)
If \( |H| = 1 \Rightarrow H = \{e\} \), a cyclic.
If \( |H| = p \text{ or } q \), by Corollary 10.11, \( H \) is cyclic.

38. Pf. Let \( \{a_i H \ | \ i = 1, \ldots, r\} \) be the collection of distinct left cosets of \( H \) in \( G \).
\( \{b_j k \ | \ j = 1, \ldots, s\} \) be the collection of distinct left cosets of \( k \) in \( H \).
Consider \( \{a_i b_j k \ | \ i = 1, \ldots, r, j = 1, \ldots, s\} \).
These are \( rs \) many left cosets of \( k \) in \( G \).
Just need to prove that they are pairwise disjoint, and cover \( G \).
\( i ) \) If \( a_i b_j k = a_i b_j k \Rightarrow a_i a_i b_j k = b_j k \)
But \( b_j k \in H \), \( b_j k \in H \Rightarrow a_i a_i b_j k \in H \)
\( \Rightarrow a_i \in a_i b_j k \Rightarrow a_i = a_i b_j k \)
Hence \( b_j k = b_j k \), but this gives \( b_j = b_j k \).
Hence for \( a_i \neq a_i k \text{ or } b_j \neq b_j k \), \( a_i b_j k \) and \( a_i b_j k \)
are different cosets of \( k \) \( \Rightarrow \) They are disjoint.
\( i b ) \forall g \in G \exists a_i \text{ s.t. } g a_i H \). \( i.e. \ a_i g = h \in H \).
Then \( \exists b_j \text{ s.t. } h \in b_j k \Rightarrow a_i g \in b_j k \Rightarrow g \in a_i b_j k \).
39. Let $H_i = G \setminus H$. Then both the left coset partition and the right cosets partition are $\{H, H_i\}$.

40. For $g \in G$. If $|\langle g \rangle| = m$, by Lagrange Thm. $m | n$.

Furthermore, $|\langle g \rangle| = m \Rightarrow g^m = e \Rightarrow g^n = e$ as well.