0.1 Math 640 — Homework 9.

1. The function \( f(i, j) = \sqrt{\frac{2}{n}} \cos((2j-1)(i-1)\frac{\pi}{2n}) \) generates an \( n \times n \) matrix \( A_n \) with orthogonal rows with \( a_{ij} = f(i, j) \). Find a multiplier diagonal \( D \) matrix that converts this matrix to an orthogonal matrix. (Hint. Think discrete cosine transform. You might also consider some numerical work to experimentally determine \( D \).)

2. Find \( A_4 \).

3. Prove or disprove: The product of Hermitian matrices is Hermitian.

4. Let \( z_1, \ldots, z_n \) be a orthonormal set of vectors in \( \mathbb{C}_n \).

   (a) Prove that if \( \alpha_1, \ldots, \alpha_n \) are real, then \( H = \sum \alpha_j z_j z_j^* \) is Hermitian.

   (b) Find the spectrum of \( H \).

   (c) Prove that \( H \) is positive definite if and only if the \( \alpha_1, \ldots, \alpha_n \) are real and positive.