Solutions to Quiz 3 (501)

1) (Note: the height of the building is irrelevant)

Partition the rope and determine the work required to
lift one piece. Let \( y_i \) = distance from the top to
the piece

\[
F = \frac{1}{2} \Delta y_i
\]

\[
S = y_i^*
\]

So

\[
W = \int_{0}^{s_0} \frac{1}{2} y \, dy
\]

\[
= \frac{1}{4} y^2 \bigg|_{0}^{s_0}
\]

\[
= 625 \text{ ft-lbs}
\]

2) Let \( F(x) = \int_{a}^{x} f(t) \, dt \).

F is differentiable and \( c \) is \( c \) since \( f \) is \( c \).

By the Mean Value Theorem, there is a \( c \in (a, b) \) such that

\[
F'(c) = \frac{F(b) - F(a)}{b - a}
\]

F'(x) = f(x) by FTC, so

\[
F'(c) = f(c)
\]

\[
F(b) = \int_{a}^{b} f(t) \, dt
\]

F(a) = \( \int_{a}^{a} f(t) \, dt = 0 \)

So

\[
f(c) = \frac{\int_{a}^{b} f(t) \, dt}{b - a}, \quad \text{or} \quad f(c)(b-a) = \int_{a}^{b} f(x) \, dx
\]

3) \( \int x \sin 4x \, dx \)

Let \( u = x \) \( \Rightarrow \) \( du = \sin 4x \, dx \)

Then \( du = dx \) \( \Rightarrow \) \( v = -\frac{1}{4} \cos 4x \)

\[
\int x \sin 4x \, dx = x \left( -\frac{1}{4} \cos 4x \right) - \int -\frac{1}{4} \cos 4x \, dx
\]

\[
= \left( -\frac{1}{4} x \cos 4x + \frac{1}{16} \sin 4x + C \right)
\]