1 7.1: Area Between Curves

We know that when $f(x) \geq 0$, on an interval $[a, b]$, then $\int_a^b f(x) \, dx$ gives the area under the graph of $f$ from $x = a$ to $x = b$.

We also know that, in general, $\int_a^b f(x) \, dx$ gives the area above the $x$-axis minus the area below the $x$-axis.

Area Between 2 Curves:

Examples:

Find the area bounded by the curves $y = x^4 - x^2$ and $y = 1 - x^2$. 
Find the area of the region bounded by \( y = \cos x \), \( y = \sin x \), \( x = 0 \), and \( x = \pi \).

Recall that the definite integral was initially defined using a partition of the \( x \)-axis and rectangles to approximate the area. However, in some cases it is possible to define the area of a region as an integral of \( y \):
Find the area of the region bounded by $x = y^2$ and $x = 3 - 2y$.

Find the area in the first quadrant to the left of $y = \ln x$ and below $y = 1$. 