1 7.2: Volume by Slicing

The volume of a prism with a base of area $B$ and a height $h$ is given by

If the height is not constant, we can, in many cases, “slice” the solid thin enough that the height is constant.

Example: Find the volume of a square pyramid whose height is $h$ and whose base is $s$ by $s$.

If the solid is generated by rotating a region about an axis, then the bases of the slices will be circles and the area will be $B = \pi r^2$. If there is a hold in the solid, we can find the volume by finding the volume of the outer solid (without the hole) minus the volume the hollowed out portion.

Examples:

Find the volume of the solid formed by rotating the region above the $x$-axis (closest to the origin) bounded by the curves $y = \sin x$ and $y = 0$ about the $x$-axis.
Find the volume of the solid formed by rotating the region bounded by \( y = 2x^2 + 1 \) and \( y = 3x \) about the \( x \)-axis.

Find the volume of the solid formed by rotating the region bounded by the curves \( y = \sqrt{x} \), \( x = 0 \), and \( y = 2 \) about the \( y \)-axis.
Find the volume of the solid formed by rotating the region in the previous example about the line $y = -1$.

The base of a solid is the ellipse $x^2 + 4y^2 = 4$. Cross-sections perpendicular to the base are squares. Find the volume of the solid.