Exam 3 REVIEW

1. Second Loan and Trust offers a 10-year certificate of deposit that earns 5.75% compounded continuously.

   a) If $10,000 is invested in this CD, how much will it be worth in 5 years?  
      Ans: $13,330.91
   b) How long will it take to be worth $25,000?  
      Ans: 15.9 years

2. How long will it take for the population of College Station to double if it continues to grow at a rate of 0.75% per year?  
   Ans: 92.4 years

3. The population of French Guiana is 199,500 (est. July 2006).
   a) If the population was 172,600 in July 2004, find the population growth rate.  
      Rate is 7.24%
   b) Name two countries that border French Guiana.  
      Look it up!

4. A note will pay $75,000 at maturity in 10 years. How much should you be willing to pay for the note now if money is worth 6.75% compounded continuously?  
   P = $38,186.73

5. Find \( f[g(x)] \) and \( g[f(x)] \) given:
   a) \( f(x) = e^{x-4} \) and \( g(x) = \sqrt{x+1} \)  
      Ans: \( f \circ g = e^{\sqrt{x+1}-4} \), \( g \circ f = \sqrt{e^{x-4}+1} \)
   b) \( f(x) = \ln |x| \) and \( g(x) = x^2 \)  
      Ans: \( f \circ g = \ln |x^2| \), \( g \circ f = (\ln |x|)^2 \)

6. Find the intervals where \( f(x) = e^{x^2+x-2} \) is increasing/decreasing.  
   Ans: increasing \((-\frac{1}{2}, \infty)\), decreasing \((-\infty, -\frac{1}{2})\)

7. Given: \( f(x) = \sqrt{8 + 4e^{-2x}} \), find \( f'(x) \).  
   Ans: \( f'(x) = \frac{-4e^{-2x}}{\sqrt{8 + 4e^{-2x}}} \)

8. Given: \( f(x) = \frac{e^{2x}}{1 + e^{2x}} \), find \( f'(x) \).  
   Ans: \( f'(x) = \frac{2e^{2x}}{(1 + e^{2x})^2} \)
9. Samantha’s Software offers hand-held computer games. If the store sells \( x \) computer games at a price of \( \$ p \) per unit, then the price-demand equation is \( p = 550e^{-0.25x} \). Find the rate of change of price with respect to demand when the demand is 15 units, and interpret.

\[ p'(15) = -3.23 \]

10. How much should Samantha’s Software charge for the hand-help games (from #9) to maximize revenue?

\[ p = 202.33 \]

11. The concentration of a certain medication in a patient’s bloodstream can be given by \( C(t) = \frac{5.3t}{t^2 + 4t + 5} \), \( 0 \leq t \leq 8 \) where \( C(t) \) is in milligrams per cubic centimeter and \( t \) is the number of hours after the medication has been administered.

a) How many hours after the medication has been administered is the concentration at a maximum?

\( t \approx 2.2 \) hours

b) What is the maximum concentration?

\( \text{Ans: } 0.63 \text{ milligrams/cubic centimeter} \)

12. \( f(x) = (\ln \ x)^3 \), find \( f'(x) \).

\[ f'(x) = \frac{3}{x} (\ln x)^2 \]

13. \( f(x) = \frac{\ln x}{x^2 + 1} \), find \( f'(x) \).

\[ f'(x) = \frac{(x^2 + 1) \cdot \frac{1}{x} - (\ln x)(2x)}{(x^2 + 1)^2} \]

14. \( f(x) = \log_8 (5x + 3x^2) \), find \( f'(x) \).

\[ f'(x) = \frac{5 + 6x}{(5x + 3x^2)\ln 8} \]

15. Find the dimensions of a closed box with a square base which has a volume of 27,000 cubic inches with minimum surface area.

\( \text{Ans: } 30” \text{ by } 30” \text{ by } 30” \)

16. A regional chain of department stores has collected the data in the table, showing weekly sales of a certain brand of jeans. The same jeans have been offered at various prices, ranging from the regular price of $35.99 to the lowest sale price of $23.99. Each pair of this brand costs the chain $20.00. Use logarithmic regression (\( p = a + b\ln x \)) to find the price (to the nearest cent) that will maximize profit.

\[ p = $27.57 \]
17. \( y = e^{5 \ln x^2} \), find \( y' \).  
\[ y' = e^{5 \ln x^2} \left[ 2 \ln x \frac{3}{x} \right] \]

18. \( y = \left( 4 - \sqrt[3]{x^2} \right)^5 \), find \( y' \).  
\[ y' = 5 \left( 4 - x^{2/3} \right)^4 \left( -\frac{2}{3} x^{-1/3} \right) \]

19. \( y = \ln |5 - x^3| \), find \( y' \).  
\[ y' = \frac{-3x^2}{5 - x^3} \]

20. Cathy’s Candies sells chocolates at a price of \( p = 4 - \frac{1}{2} \ln x \), when \( x \) pieces are purchased. If the current price is 75¢ each, should she increase or decrease the price to increase revenue?  
\[ \text{Ans: The price should be decreased, since } E(.75) = 1.5. \]

21. Given the price-demand function, \( p + .01x = 40 \),
   a) find \( E(p) \).  
\[ \text{Ans: } E(p) = \frac{p}{40 - p} \]
   b) Is \( E(15) \) elastic, or inelastic and should the price be raised or lowered?  
\[ \text{Ans: increased} \]
   c) Find the price for which there is unit elasticity.  
\[ \text{Ans: } p = \$20. \]

22. \( \int \sqrt[4]{x^5} \, dx = \)  
\[ \text{Ans: } \frac{4}{9} x^{9/4} + C \]

23. \( \int \left( \sqrt{x} - \frac{1}{\sqrt{x}} \right) \, dx = \)  
\[ \text{Ans: } \frac{2}{3} x^{3/2} - \frac{3}{4} x^{3/2} + C \]

24. \( \int \frac{x^3 + x^5}{x^4} \, dx = \)  
\[ \text{Ans: } \ln x + \frac{1}{2} x^2 + C \]
25. \[ \int (e^x + e) \, dx = \quad \text{Ans: } e^x + e + C \]

26. Find the cost function if the marginal cost in dollars is given by \( MC = 28x - 10e^x \) where \( x \) is the number of items sold and there are fixed cost of $50.
   \[ C(x) = 14x^2 - 10e^x + 60 \]

27. The population of Smallsville, Ohio is increasing at the rate \( 2400e^{0.02t} \) where \( t \) is the number of years since 1950, when the population was 30,000. Find the population in 1985 according to this model. \[ P(35) \approx 151,650 \]

28. \[ \int (x^2 - 2)(x+5) \, dx = \quad \text{Ans: } \frac{1}{4}x^4 + \frac{5}{3}x^3 - x^2 - 10x + C \]

29. Find the revenue function if the marginal revenue is \( R'(x) = 625 - \frac{2}{5}x \).
   \[ R(x) = 625x - \frac{1}{5}x^2 \]

30. \[ \int e^3 \, dx = \quad \text{Ans: } e^3x + C \]

31. \[ \int (5t^{-1} + 1) \, dt = \quad \text{Ans: } 5\ln|t| + t + C \]

32. \[ \int 14x^{3/4} \, dx = \quad \text{Ans: } 8x^{7/4} + C \]

33. \[ \int 2e^{3x} \, dx = \quad \text{Ans: } \frac{1}{2}e^{4x} + C \]

34. \[ \int \frac{5x}{x^2 + 1} \, dx = \quad \text{Ans: } \frac{5}{2}\ln|x^2 + 1| + C \]

35. \[ \int \frac{1}{x \ln x} \, dx = \quad \text{Ans: } \ln|\ln|x|| + C \]

36. \[ \int \frac{1}{x(\ln x)^2} \, dx = \quad \text{Ans: } \frac{-1}{\ln|x|} + C \]

37. \[ \int 4t(t-1)^{15} \, dt = \quad \text{Ans: } \frac{4}{17}(t-1)^{17} + \frac{1}{4}(t-1)^{16} + C \]

38. \[ \int \frac{t-5}{(t+1)^2} \, dt \quad \text{Ans: } \ln|t+1| + \frac{6}{t+1} + C \]
39. Given \( y = \ln | \log_4 |4x+5| \) , find \( y' \).  
Ans: \( \frac{4}{(4x+5)(\ln 4)\log_4 |4x+5|} \)

40. Find the exact area between the x-axis and the graph of \( f(x) = \sqrt{9-x^2} \) over the interval from \( x = 0 \) to \( x = 3 \).  
Ans: \( \frac{9}{4} \pi \) square units

41. Given: \( \int_2^5 x \, dx = 10.5, \int_2^5 x^2 \, dx = 39, \int_5^8 x^2 \, dx = 129 \), evaluate:

\[ a) \int_2^5 (3x^2 - 5x) \, dx \hspace{1cm} b) \int_2^8 -3x^2 \, dx \]

Ans: 64.5  
Ans: -504

42. During a special study on learning, a psychologist found that, on the average, the rate of learning a list of special symbols in a code, \( N'(x) \), after \( x \) days of practice was given approximately by the following table values:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N'(x) )</td>
<td>32</td>
<td>29</td>
<td>25</td>
<td>21</td>
<td>18</td>
<td>16</td>
<td>12</td>
</tr>
</tbody>
</table>

Approximate the area under the graph of \( N'(x) \) from \( x = 4 \) to \( x = 10 \) using left and right sums over three equal subintervals.  
Ans: \( A_L = 128, A_R = 110 \)

*From: p417, #59, Barnett: Calculus for Business, Economics, Life Sciences and Social Sciences*

43. Evaluate each of the following:

\[ a) \int_{-2}^{2} (4-x^2) \, dx \hspace{1cm} b) \int_{1}^{5} e^{2x} \, dx \]

Ans: \( 10 \frac{2}{3} \)  
Ans: \( \frac{1}{2} e^{10} - \frac{1}{2} e^2 \)

\[ c) \int_{0}^{1} x \cdot e^{x^2+1} \, dx \hspace{1cm} d) \int_{0}^{1} \frac{1}{\sqrt{3x+1}} \, dx \]

Ans: \( \frac{1}{2} e^2 - \frac{1}{2} e \)  
Ans: \( \frac{2}{3} \)
44. Refer to the figure below of \( F'(x) \). If \( F(0) = 2 \), what is \( F(4) \)? What is \( F(8) \)?

\[ y = F'(x) \]

Ans: \( F(4) = 22 \), \( F(8) = 32 \)

44. The Scandi-Trac Company determines that their marginal profit function for producing and selling a new economy model of cross-country ski machine at a mall is given by \( MP(x) = P'(x) = 0.3x^2 + 0.2x \), \( 0 \leq x \leq 30 \) where \( x \) is the number of machines produced and sold and \( P'(x) \) is the marginal profit function measured in dollars per ski machine.

a) Knowing that $704 profit is made when 20 ski machines are sold, find the profit function \( P(x) \).

Ans: \( P(x) = .1x^3 + .1x^2 - 136 \)

b) Evaluate: \( \int_{20}^{10} P'(x) \, dx \) and interpret.

Ans: The profit from the last 10 ski machines when 20 are produced and sold is $730.

45. \( \int 6(x + 1)\left(2x^2 + 4x - 5\right)^{3/2} \, dx \)

Ans: \( \frac{3}{5}(2x^2 + 4x - 5)^{5/2} + C \)

46. \( \int 5x(3x^2 + 10)^{1/2} \, dx \)

Ans: \( \frac{5}{9}(3x^2 + 10)^{3/2} + C \)

47. \( \int \frac{x}{\sqrt{x^2 - 1}} \, dx \)

Ans: \( \frac{3}{4}(x^2 - 1)^{3/2} + C \)

48. \( \int_{-5}^0 \frac{\sqrt{2} - x}{\sqrt{2} + x} \, dx \)

Ans: \( \frac{3}{4}(7)^{3/2} - \frac{3}{4}(2)^{3/2} \)

49. \( \int \frac{6x^2 - 6}{x^3 - 3x + 4} \, dx \)

Ans: \( 2\ln|x^3 - 3x + 4| + C \)

50. \( \int \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}} \, dx \)

Ans: \( \frac{1}{2}\ln|e^{2x} - e^{-2x}| + C \)
51. If supply and demand for a product are given by \( p = 4 + \frac{1}{4} x^2 \) and \( p = 16 - \frac{1}{2} x^2 \) respectively, find the following at equilibrium price:

   a) consumers’ surplus  
   Ans: \( 21 \frac{1}{3} \)

   b) producers’ surplus  
   Ans: \( 10 \frac{2}{3} \)

52. The value of the house is given by \( V(t) = 150,000 e^{0.04t}, \ 0 \leq t \leq 25 \), where \( t \) represents the number of years since it was built in 2005, and \( V(t) \) represents the value in dollars. Find the average value of the house during the first ten years.
   Ans: \$184,434.26

Ogden, Utah