**Math 142 Lecture Notes**  
**Section 1.4 – Quadratic Functions**

**Quadratic Functions:**

**Definitions:**
1) a second-degree function

2) \( y = x^2 \) symmetry about the y-axis

3) lowest point on the graph is at the origin: \((0,0)\)

**Parabolas:** Use the rules we introduced for transformations of a basic curve and describe each of the following functions.

1. \( f(x) = 2(x - 1)^2 \)

2. \( g(x) = -3(x + 5)^2 \)

3. \( h(x) = \frac{1}{2}(x - 4)^2 \)

4. \( F(x) = -6(x - 3)^2 + 2 \)

5. \( G(x) = \frac{1}{a}(x + b)^2 - c \), where \( a, b, c \in I^+ \)
Quadratic Functions

If \( a, b, \) and \( c \) are real numbers with \( a \neq 0 \), then the function
\[
f(x) = ax^2 + bx + c
\]
is a **quadratic function** and its graph is a **parabola**.

1) domain: __________________

2) range: ___________________

**Intercepts of a parabola:**

\[ y = ax^2 + bx + c \]

1) x-intercept:

2) y-intercept:

**Quadratic formula**

If \( ax^2 + bx + c = 0, a \neq 0 \), then
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

if \( b^2 - 4ac \geq 0 \).

**Example:**

\[ f(x) = 3x^2 + 30x + 75 \]

1) Graph

2) Find the x- and y-intercepts.

3) Solve \( 3x^2 + 30x + 75 \geq 0 \) graphically.

**Standard form:**

\[ y = ax^2 + bx + c \quad , \quad a \neq 0 \]

**Vertex form:**

\[ y = a(x - h)^2 + k \quad \text{where} \quad (h, k) \quad \text{is the vertex}. \]

Where is the vertex?
Where is the axis of symmetry?
What is the maximum value?
Complete the square and transform the following function into vertex form:

\[ y = 5x^2 - 40x + 87 \]

Describe the graph of the function \( y = 5x^2 - 40x + 87 \).

**Note:** For parabolas that open up/down, with the vertex at \((h,k)\), the axis of symmetry is the line \(x = h\). For parabolas that open right/left, with the vertex at \((h,k)\), the axis of symmetry is the line \(y = k\).

Find the axis of symmetry for the following graph:

What is the domain?__________    What is the range?__________

Find the equation of the parabola shown above.

**Vertex:** The maximum or minimum point on a parabola that opens up or down.  \( \text{Vertex} = (h, k) \)
Example: \[ f(x) = 0.5x^2 - 4x + 12 \]

Find the vertex form for \( f(x) \).

Find the vertex and the maximum or minimum.

Find the range.

Describe the graph.

Applications

A survey of shops at the mall, shows the following data:

- The price demand function for making \( x \) items is \( 96 - 3x \).
- The cost to make \( x \) items is \( 165 + 48x \).

Find the break-even point(s) given that \( x \) represents hundreds of circuits, and cost is measured in thousands of dollars.

What is the wholesale price per circuit (rounded to the nearest dollar) that produces the maximum revenue?