Math 142 Lecture Notes
Section 2.1 – Polynomial and Rational Functions

Polynomial Functions: A function that can be written in the form:
\[ f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \]
where \( n \) is a nonnegative integer, called the degree of the polynomial. It is the highest exponent present in the term. The domain is the set of real numbers. \( a_n \neq 0 \)

Important points:
1) The shape is connected to the degree of the polynomial.
2) The sign (+/-) of the leading coefficient describes the end-line behavior.
3) Examples of even and odd polynomials:

![Graphs of polynomial functions](image)

from: pg 80 of Calculus for Business, Economics, Life Sciences and Social Sciences by: Barnett, Ziegler and Byleen

4) Even functions:
   - start high and end high, or start low and end low depending on the sign of the leading coefficient

5) Odd functions:
   - start high and end low, or start low and end high depending on the sign of the leading coefficient

6) Graphs of polynomials are **continuous**. No holes or breaks, and NO sharp corners.

7) **Turning point**: a place on the graph where the curve changes from increasing to decreasing or vice versa.
Section 2-1

Theorem 1  The graph of a polynomial function of positive degree n can have at most \( n - 1 \) turning points. It can cross the x-axis at most n times.

Answer the following questions:
1. What is the least number of turning points an odd-degree polynomial function can have?
2. What is the least number of turning points an even-degree polynomial function can have?
3. What is the maximum number of x-intercepts for a polynomial of n\textsuperscript{th} degree?
4. What is the least number of x-intercepts for a polynomial of n\textsuperscript{th} degree,
   a. if the polynomial has odd degree?
   b. if the polynomial has even degree?
5. What is the least number of real solutions of a polynomial function
   a. of odd degree?
   b. of even degree?

Regression Polynomials:
1. Estimate the value of the car after by finding a quadratic function which relates it’s age and value:

<table>
<thead>
<tr>
<th>Age in yrs</th>
<th>0</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value in $'s</td>
<td>42,120</td>
<td>39,575</td>
<td>36,500</td>
<td>28,455</td>
<td>18,750</td>
<td>12,575</td>
<td>4,580</td>
</tr>
</tbody>
</table>

A. value after 4 yrs: ___________
B. value after 10 yrs: ___________
C. What is the quadratic function you used to find the value of the car? ________________

Rational Functions
A rational function is any function that can be written in the form
\[ f(x) = \frac{n(x)}{d(x)}, \quad d(x) \neq 0 \]  and \( n(x) \) and \( d(x) \) are polynomials.

Domains: Check for values that make the denominator zero, or any values that would make an even-indexed radical negative, or if logarithms are present, the values needed to keep the expression positive, and if it’s a word problem, check to see what values make sense in the problem.

A value which would make the denominator zero is point of discontinuity.

Intercepts: Let \( x=0 \) and solve for \( f(x) \), then let \( f(x)=0 \) and solve for \( x \).
Asymptotes: A line the graph approaches. Asymptotes are graphed with dotted lines.

**Vertical asymptotes** If \( a \) is a real number, such that \( d(a)=0 \), then \( x=a \) is a vertical asymptote.

**Horizontal asymptotes** Compare the degrees of the numerator and the denominator.

\[
f(x) = \frac{ax^n + \ldots + c}{bx^d + \ldots + f}
\]

Note: in this example the degree of the numerator is \( n \), and the degree of the denominator is \( d \).

1) if \( n>d \), then no horizontal asymptote exist
2) if \( n=d \), then a horizontal asymptote exist at \( y = \frac{a}{b} \)
3) if \( n<d \), then a horizontal asymptote exist at \( y=0 \)

Note: You can always divide every term in the numerator and the denominator, by the highest power present in the denominator, to change it into a simpler form and then determine the asymptotes.

**Oblique asymptotes** If the degree of the numerator is ONE more than the degree of the denominator, then an oblique (or slanted) asymptote exists. Discussed more in Ch 4

**Graph:**

\[
f(x) = \frac{4-4x}{2x-10}
\]

1) Intercepts:
2) Vertical asymptote:
3) Horizontal asymptote:
4) Graph: