Math142 Lecture Notes
6.2 - Integration by Substitution

FIND $\frac{d}{dx} (3x^2 + 2x)^3$

ANSWER $3(3x^2 + 2x)^2(6x + 2)$

SO HOW DO WE FIND $\int [3(3x^2 + 2x)^2(6x + 2)] \, dx$

In general, if we can express the integral as

$$\int (\text{expression})^\text{power} \cdot (\text{expression})' \, dx$$

we can use $u$-substitution, and write the question and answer as

$$\int (u)^n \cdot u' \, du = \frac{1}{n+1} u^{n+1} + C$$

Then replace $u$ with it’s equivalent expression.

Note: Answers can be checked by taking the derivative.

Example 1: Evaluate the following.

(a) $\int 8(8x - 1)^2 \, dx$

(b) $\int 8t^3 \sqrt{4t^2 - 15} \, dt$

(c) $\int \frac{6x^2 - 1}{(2x^3 - x)^4} \, dx$
Sometimes you will need to multiply in a constant to obtain the form

\[ \int (\text{expression})^\text{power} \cdot (\text{expression})' \, dx \]

Example 2: Evaluate each of the following.

(a) \( \int t^2(t^3 - 1)^3 \, dt \)

(b) \( \int \frac{x^2}{(2x^3 - 5)^4} \, dx \)

(c) \( \int (10x + 10)(x^2 + 2x - 7)^3 \, dx \)

(d) \( \int e^{-5x} \, dx \)

(e) \( \int \frac{8x}{x^2 - 10} \, dx \)
Solving the \( u \)-expression for \( x \) to Complete a Substitution

Example 3: Find \( \int x \sqrt{x - 2} \, dx \)

Example 4: Find \( \int t e^{-t^2} \, dt \)

Example 5: Find \( \int 2x^3 \sqrt{x^4 + 5} \, dx \)

Suggestions:

- Always write the expression with the exponent first. \( c \int (\text{mess})^#(\text{mess})' \, dx \)
- Move constants outside (in front of) the integral sign.
- Remember: you can multiply by a constant, but not by a variable.
- Change all radicals to exponential form.
- Look for the derivative of the expression: \( (expression)' \) or \( (mess)' \)
- When you have that, you are ready to integrate!