Math142 Week In Review # 2

The Most Important Problems to Understand - This Week

1. Find the average rate of change between the following sets of points:
   a. \((8, -3)\) and \((5, 2)\) \hspace{1cm} \text{ans: } -\frac{5}{3}
   b. \((a, b - 1)\) and \((a - 1, b + 1)\) \hspace{1cm} \text{ans: } -2

2. Write the equation of the line passing through the point \((5, -2)\) that:
   a. has an x-intercept of 2. \hspace{1cm} \text{ans: } 2x + 3y = 4
   b. that passes through the origin. \hspace{1cm} \text{ans: } 2x + 5y = 0
   c. that is horizontal. \hspace{1cm} \text{ans: } y = -2

3. The Maximum Monitor priced the 18” sets for $2400 and sold 32. The following weekend they moved the price to $2375 and sold 35. Find a price-demand function which fits this model.
   \hspace{1cm} \text{ans: } p = -\frac{25}{3}x + \frac{8000}{3}

4. A new DVD player costs $200 and in 2 years is worth $185.
   a. What is the equation for this depreciation function, assuming it is linear?
      \hspace{1cm} \text{ans: } V = -7.5t + 200
   b. What is the value of the DVD player in six years?
      \hspace{1cm} \text{ans: } $155
   c. What is the life expectancy of this machine?
      \hspace{1cm} \text{ans: } t = 26\frac{2}{3} \text{ yrs}

5. Make an accurate graph for each of the following:
   a. \(f(x) = \begin{cases} |x + 5|, & x \leq 4 \\ x^2 - 30, & x > 5 \end{cases}\)
   b. \(g(x) = \begin{cases} -2, & x < -3 \\ x + 1, & -3 \leq x < 9 \\ \sqrt{x}, & x \geq 9 \end{cases}\)

6. Write a piece-wise defined function to represent the function above.
   \hspace{1cm} \text{ans: } f(x) = \begin{cases} -x + 2, & x < -4 \\ 12, & -4 \leq x \leq 0 \\ |x - 8| - 2, & x > 2 \end{cases}

7. The amount spent annually in college bookstores in the U.S. is modeled by
   \(f(x) = 0.19x + 1.6\)
   where \(x\) is the number of years since 1982, and \(f(x)\) is the amount spent in billions of dollars.
   a. How much is the spending increasing each year? \hspace{1cm} \text{ans: } $190,000,000/year
   b. According to this model, how much was spent in 1990? \hspace{1cm} \text{ans: } $3.12 billion

8. Given: \(P(x) = 3x^2 - 39x + 120,\)
   a. find the intervals over which the function is increasing and decreasing.
      \hspace{1cm} \text{ans: increasing } (6.5, \infty) \text{ and decreasing } (-\infty, 6.5)
   b. find the axis of symmetry. \hspace{1cm} \text{ans: } x = 6.5

9. Using the graph of \(f(x)\) below, sketch the graph of each of the following functions.
   a. \(y = -f(x - 2)\)
   b. \(y = 2f(x) + 3\)
10. Write the equation of the parabola which opens up, passes through the point \((-4, 8)\) and has a vertex at \((1, -2)\).

\[ y = \frac{2}{3} (x - 1)^2 - 2 \]

11. Given \(p(x) = 105.7 - 0.89x\) and variable costs are \$80/unit and fixed costs are \$61.80.

a. Find the cost equation.

\[ C = 80x + 61.80 \]

b. Find the revenue equation.

\[ R = 105.7x - 0.89x^2 \]

c. Find the profit equation.

\[ P = 25.7x - 0.89x^2 - 61.80 \]

d. Find the break even points.

\((2.67, 2160.12), (26.23, 2160.12)\)

12. Determine if each of the following is a polynomial:

a. \(f(x) = 5x^2 - \pi x + 4\)  
   \(\text{ans: yes}\)

b. \(g(x) = 3x - \frac{2}{x-3} + 10\)  
   \(\text{ans: yes}\)

c. \(h(x) = \sqrt{16x^4}\)  
   \(\text{ans: no}\)

d. \(F(x) = 3ix^2 + 9\)  
   \(\text{ans: no}\)

13. Determine the end line behavior of each of the following:

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x) = 4x^3 - 6x + 4)</th>
<th>(\text{as } x \to \infty)</th>
<th>(\text{as } x \to -\infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>(f(x) = 4x^3 - 6x + 4)</td>
<td>(\infty)</td>
<td>(-\infty)</td>
</tr>
<tr>
<td>b.</td>
<td>(g(x) = -6x^3 - 4x + 2)</td>
<td>(-\infty)</td>
<td>(-\infty)</td>
</tr>
<tr>
<td>c.</td>
<td>(h(x) = -2x^3 + 7x + 12)</td>
<td>(-\infty)</td>
<td>(\infty)</td>
</tr>
</tbody>
</table>

14. Given the function \(g(x) = \frac{3x^2 + 5x - 2}{3x^2 - 10x + 3}\),

a. find any holes, vertical asymptotes, or horizontal asymptotes.

\(\text{ans: hole at } x = \frac{1}{3}, \text{ VA at } x = 3, \text{ HA at } y = 1\)

b. find any intercepts. \(\text{ans: } (-2, 0), (0, -\frac{2}{3})\)

c. graph.

15. Given the function \(f(x) = \frac{2x^2 + 10x + 12}{x^2 - 9}\),

a. find any holes, vertical asymptotes, or horizontal asymptotes.

\(\text{ans: hole at } x = -3, \text{ VA at } x = 3, \text{ HA at } y = 2\)

b. find any intercepts. \(\text{ans: } (0, -\frac{4}{3}), (-2, 0)\)

c. graph.

16. Given the function \(h(x) = \frac{6 - 3x}{x - 6}\),

a. find any holes, vertical asymptotes, or horizontal asymptotes.

\(\text{ans: no holes, VA at } x = 6, \text{ HA at } y = -3\)

b. find any intercepts. \(\text{ans: } (0, -1), (2, 0)\)

c. graph.

17. The Medical Diagnostic Lab, Inc. has developed a new low cost test for heartworm in dogs, and promotes sales through a sales campaign. The income from sales is given by

\[ S(x) = \frac{50x^2 - 200x + 9}{2x^2 + 10x + 1} \quad x \geq 5 \]

where \(x\) represents the number of thousands of dollars spent on advertising and \(S(x)\) represents the income from sales in tens of thousands of dollars.

a. Evaluate \(S(8)\) and interpret.

\(\text{ans: } S(8) = 7.6985646\), When $8,000 is spent on ads, the result is $76,985.65 income from sales.

b. Find the horizontal asymptote and interpret.

\(\text{ans: } y = 25\), With additional dollars spent on ads, the income from sales approaches $250,000.