MATH 150 Lecture Notes for Section 1.1

Real Numbers

- Natural #’s: $N = \{1, 2, 3, 4, \ldots\}$
- Whole #’s: $W = \{0, 1, 2, 3, 4, \ldots\}$
- Integers: $I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- Rational #’s: $R = \left\{\frac{m}{n} \mid m, n \in I, n \neq 0\right\}$
- Irrational #’s: non-repeating, non-terminating decimal number

ALL of these sets, together, make up the set of Real Numbers, $\mathbb{R}$.

To change repeating decimals into rational form:
- multiply by $10^n$ where $n$ is the number of digits that repeat
- ie: Suppose $x = .\overline{7123}$
- Write $x$ in fraction form.

PROPERTIES OF REAL NUMBERS:

1. The Commutative Properties
   - addition: $a + b = b + a$
   - multiplication: $ab = ba$

2. The Associative Properties
   - addition: $a + (b + c) = (a + b) + c$
   - multiplication: $a(bc) = (ab)c$

3. The Distributive Property
   - $a(b + c) = ab + ac$

4. Identities - an identity is a unique number where the answer for a particular operation is the original number
   - for addition the identity element is 0: $a + 0 = 0 + a = a$
   - for multiplication the identity element is 1: $a \cdot 1 = 1 \cdot a = a$

5. Inverses - an inverse is a number which when acted with the original number, results in the identity
• Each real number $a$ has a unique **additive inverse** represented by $-a$:

$$a + (-a) = (-a) + a = 0$$

• Each **nonzero** real number $a$ has a unique **multiplicative inverse** represented by $\frac{1}{a}$:

$$a \left( \frac{1}{a} \right) = \left( \frac{1}{a} \right) a = 1$$

**Working With Fractions**

• Multiplying Fractions

• Dividing Fractions

• Adding Fractions

• If denominators are different:

• Reducing Fractions

• Cross Products

• Using the LCD to add fractions

**The Real Line**

• Always label units on the graph.
Inequalities For $a, b \in R$, one and only one of the following holds:

$$a < b, \ a > b, \ or \ a = b$$

Sets and Intervals: A set is a well defined collection of objects. The objects in the set are called elements. Sets are described by one of the following methods:

- roster method
- set builder notation
- graph

Symbols

- $\in$: is an element of
- $\notin$: is not an element of

Union:

$A \cup B$ is the set which contains all elements in $A$ or $B$ or both.

Intersection:

$A \cap B$ is the set which contains all elements in common, elements in both $A$ and $B$.

- The empty set, $\emptyset$, contains no elements.

Example: Given $Q = \{a, b, c, d, e, f\}$ and $R = \{a, e, i, o, u\}$

- find: $Q \cup R$

- find: $Q \cap R$

Interval Notation: the following notation represents different intervals

<table>
<thead>
<tr>
<th>open interval</th>
<th>Interval Notation</th>
<th>Inequality Notation</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(a, b)$</td>
<td>$a &lt; x &lt; b$</td>
<td></td>
<td><img src="https://via.placeholder.com/150" alt="Diagram" /></td>
</tr>
<tr>
<td>closed interval</td>
<td>$[a, b]$</td>
<td>$a \leq x \leq b$</td>
<td><img src="https://via.placeholder.com/150" alt="Diagram" /></td>
</tr>
</tbody>
</table>
Example: Graph each of the following given $A = (-2, 8)$, $B = (-\infty, -1)$, and $C = [5, \infty)$

- graph A

- graph B

- graph C

- find $A \cap B$

- find $A \cup B$

- find $A \cap C$

- find $A \cup C$

- find $B \cap C$

- find $B \cup C$
**Absolute Value:** definition

\[ |x| = \begin{cases} 
  x & \text{if } x \geq 0 \\
  -x & \text{if } x < 0 
\end{cases} \]

- Find the absolute value of each of the following:
  1. \( |7 - 4| \)
  2. \( |2 - \pi| \)
  3. \( |5 - \sqrt{3}| \)
  4. \( |2 - \sqrt{7}| \)
  5. \( |\triangle - \star| \)

**Distance between two points on a number line**

- The distance between \(a\) and \(b\) is given by \(|a - b|\) or \(|b - a|\)

\[ d(a, b) = |b - a| = |a - b| \]