150 Lecture Notes for Section 1.3
Algebraic Expressions

**variables:** a letter or symbol whose value changes

\[ x = \{ \text{my age while a college Student} \} \]
\[ x = \{17, 18, 19, 20, 23\} \]

**domain** is the set of numbers the variable can be replaced with Is 10 in the domain of \(x\), as defined above?

**variables:** \(x, y, z, w, s, t\) are the letters commonly used

**constant:** a fixed number

5, \(\pi\), \(\sqrt{3}\), \(a, b, c\), are examples of constants

**Algebraic Expressions** contain variables, constants, and symbols representing the operations: addition, subtraction, multiplication, division, powers and roots.

\[
\frac{cx^2y + dy^2z}{\sqrt{x^2 + y^2 + z^2}}
\]

**A polynomial** is an expression of the form

\[ a_n x^n + a_{n-1} x^{n-1} + ... + a_2 x^2 + a_1 x + a_0 \quad \text{where} \quad a_n \neq 0 \]

where the \(a_i\)’s are real numbers, \(x\) is the variable, and \(n\) is a nonnegative integer called the **degree** of the polynomial. Each expression \(a_i x^i\) is called a **term** of the polynomial.

The simplest algebraic expressions use only +, −, \(x\), with exponents that are non-negative integers.

To classify polynomials use the following terms:

- 1 term **monomial** 3\(x^4\)
- 2 terms **binomial** 2\(x^2 + 1\)
- 3 terms **trinomial** \(x^2 + 4x + 5\)

Review: simplifying expressions by combining **like terms**: same variables raised to the same power.

**Domains:** are all real numbers, \(\mathbb{R}\), unless:

- (1) the expression is a fraction
  in that case, exclude values which make the denominator zero
- (2) the expression is an even-indexed radical
  in that case, set the quantity under the radical \(\geq 0\), and **solve**
• (3) the expression contains a logarithm
set the quantity > zero
• (4) application problems
only allow numbers which make sense.

Examples:

Special Product Formulas:
• \((A - B)(A + B) =\)
• \((A + B)^2 =\)
• \((A - B)^2 =\)
• \((A + B)^3 =\)
• \((A - B)^3 =\)

Use the special product formula to simplify each of the following:
• \((x^2 - \sqrt{y})(x^2 + \sqrt{y}) =\)
• \((a^3 + 4b)^2 =\)
• \((x^2 + 2)^3 =\)

FACTOR - To factor a polynomial is to rewrite it as a product of polynomials. There are six main ways to factor polynomials:
• look for a common factor \textbf{first}
• difference of 2 squares:
• difference of 2 cubes:
• sum of 2 cubes:
• factor by trial and error (backwards foil)
• factor by grouping

Examples