Math150 Lecture Notes
3.1 - Polynomial Functions and Their Graphs

Definition - Polynomial Function
A polynomial function of degree \( n \) has the form
\[
f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0
\]
where \( n \) is a nonnegative number and \( a_n \neq 0 \).

Example 1: Name the degree of each of the following polynomials:
(a) \( P(x) = 3x^2 - 4x + 9 \)
(b) \( Q(x) = 3x^2 - 4x^3 + 2 \)
(c) \( S(x) = 4x^{-1} + 2x + 5 \)

Graphs of Polynomials

Types of Polynomials:
(a) monomials
(b) binomials
(c) trinomials

End Behavior of Polynomials
The end behavior is determined by the degree \( n \) and the sign of the leading coefficient \( a_n \).
\[
y = P(x) \text{ has odd degree} \\
y = P(x) \text{ has even degree}
\]
Using Zeros to Graph Polynomials

**Zeros of Polynomials**

If $P$ is a polynomial and if $c$ is a number such that $P(c) = 0$, then $c$ is a zero of $P$. This also means:

- $c$ is a zero of $P$.
- $x = c$ is a root of the equation $P(x) = 0$.
- $x - c$ is a factor of $P(x)$.

Example 3: Given $P(x) = 3x^4 - 48$; factor the expression into

$$P(x) = 3(x - 2)(x + 2)(x^2 + 4)$$

$2$ and $-2$ are zero of $P$.

$x = 2, x = -2$ are roots of the equation $P(x) = 3x^4 - 48$.

$x - 2$ and $x + 2$ are factors of $3x^4 - 48$.

$x = 2, -2$ are x-intercepts on the graph of $P$.

**Intermediate Value Theorem for Polynomials**

If $P$ is a polynomial function and $P(a)$ and $P(b)$ have different signs, + and −, then there exists at least one value $c$ such that $P(c) = 0$

Example 4: Graph: $P(x) = x^4(x - 3)(x + 5)^2$

(a) Determine the degree of the polynomial.

(b) Determine the zeros.

(c) Determine the end line behavior.

(d) Sketch the graph.

The number of local extrema is at most, one less than the degree $n$ of the polynomial.