Math150  Lecture Notes  4.1

Exponential Functions

\( f(x) = a^x \) is an exponential function with base \( a \), where \( a \) is positive and not equal to 1

Graph:: \( f(x) = 3^x \) \hspace{1cm} \( f(x) = 2^x \)

domain: \( \mathbb{R}, (-\infty, \infty) \)
range: \( (0, \infty) \)
intercept: \( (0, 1) \)
increasing
\( y = 0 \) is a horizontal asymptote
continuous

\[
\begin{align*}
f(x) &= 3^x - 1 \\
g(x) &= 3^x + 2 \\
f(x) &= 3^{x+1}
\end{align*}
\]

\[
\begin{align*}
f(x) &= 2^x \\
g(x) &= -2^x
\end{align*}
\]

\[
\begin{align*}
f(x) &= 2^x \\
h(x) &= 2^{-x}
\end{align*}
\]

\[ e \approx 2.71828... \quad \text{natural base} \]
\[ f(x) = e^x \quad \text{natural exponential function} \]

\[ (1 + \frac{1}{x})^x \to e \text{ as } x \to \infty \]

\( y = a^{-x} \) is an exponential function with base \( a \)

Graph: \( f(x) = 2^{-x} \quad f(x) = 3^{-x} \)

- \( \text{domain: } \mathbb{R} \quad (-\infty, \infty) \)
- \( \text{range: } (0, \infty) \)
- \( \text{intercept } (0, 1) \)
- \( \text{decreasing} \)
- \( y = 0 \) is a horizontal asymptote
- \( \text{continuous} \)

\[ f(x) = 2^{-x} - 1 \quad g(x) = 2^{-x} + 2 \quad h(x) = 2^{-x+1} \]

**Compound Interest**

\[ A = P(1 + \frac{r}{n})^{nt} \]

- \( P = \text{principal} \)
- \( r = \text{interest rate expressed as a decimal} \)
- \( n = \# \text{ of times compounded in ONE year} \)
- \( t = \# \text{ of years} \)

**Continuously Compounded Interest**

\[ A = Pe^{rt} \]
Example 1: A sum of $5000 is invested at an annual rate of \(7\frac{3}{4}\%\), compounded quarterly. Find the balance after 6 years.

\[
P = 5000 \quad A = 5000\left(1 + \frac{0.0775}{4}\right)^{4(6)}
\]

\[
r = 0.0775 \quad n = 4 \quad A \approx 7,924.75
\]

\[
t = 6
\]

Example 2: A sum of $12,000 is invested at an annual rate of \(6\frac{1}{2}\%\), compounded weekly. Find the balance in 8 months.

\[
A = 12,000\left(1 + \frac{0.065}{52}\right)^{52(8)} = 12,531.09
\]

Example 3: $1000 is invested at 12% per year. Find the amount in the account after 3 years, if the interest is compounded:

a) quarterly  

\[
A \approx 1,425.76
\]

b) monthly

\[
A \approx 1,430.77
\]

c) weekly

\[
A \approx 1,432.74
\]

**Calculator Solution:** Finance, 1:TVM Solver

\[
N = n \times t = (\text{number time calculated in one year}) \times (\text{number of years})
\]

I = rate as a %

PV = present value

PMT = payments

FV = future value

P/Y = payments/year (same as little "n")

C/Y = calculations/year (same as little "n")

PMT = END Begin

A sum of $1000 is invested at 12% for 3 years. Find the future value if compounded:

a) quarterly  

\[
A \approx 1,425.76
\]

b) monthly

\[
A \approx 1,430.77
\]

c) weekly

\[
A \approx 1,432.74
\]

Continuously Compounded Interest

\[
A = Pe^{rt}
\]

$2500 is invested at \(7\frac{1}{2}\%\) compounded continuously for 8 years. Find the accumulated amount.